Signals-From-Noise
What Sallen-Key Filter Articles Don't Tell You
Part III: The ac constraints that you need to know
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Last month's column http://www.analogzone.com/iot_051407.htm showed the effect that op amp bandwidth and output impedance has on Sallen-Key filter performance. The remaining effect to consider is noise.

For a better understanding of noise in op amp circuits I recommend:
- Linear Technology's Design Note 15 (**Noise Calculations in Op Amp Circuits**) http://www.linear.com/pc/downloadDocument.do?id=4201
- The Five part TechNote in analogZONE by TI's Art Kay (*Analysis and Measurement of Intrinsic Noise in Op Amp Circuits*) http://www.analogzone.com/avt_043007.htm

By now, it should be pretty obvious that Sallen-Key filter noise performance is affected by component selection. The three types of noise sources are:
- Op amp voltage noise
- Op amp current noise
- Resistor thermal noise

Each requires different techniques to reduce noise. A noise model of a Sallen-Key low-pass filter is:

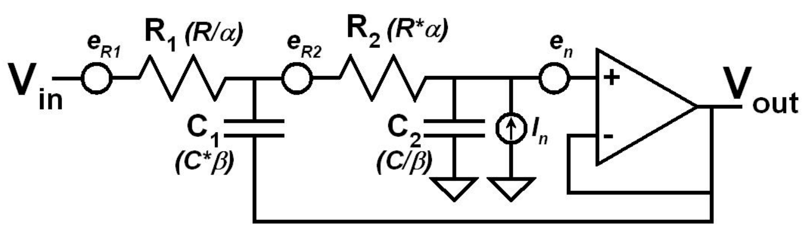

Fig. 1: Basic Sallen-Key Low-Pass Filter Noise Model

Using superposition, each noise source will be singularly analyzed and the results summed to produce the total noise.

Op Amp Voltage Noise

Op amp noise is measured relative to its input (**rti**) with performance determined by its spectral noise density and 1/f noise corner frequency (**f_c**). Spectral noise is flat and is measured in volts per root hertz (**√Hz**). The 1/f noise density increases by the inverse of the frequency. The point where these noise values match is the corner frequency. The noise density above this corner frequency is flat while below the noise increases. 1/f noise is sometime called *flicker* and *wander*. For low-frequency filter applications this 1/f noise may be dominant. At low frequencies an op amp with a noise density of 2 nV/√Hz and a roll off frequency of 1 kHz, may produce more overall noise than an op amp with 10 nV/√Hz noise density and a 10 Hz roll-off frequency. These two values are a function of transistor type, process, and design. And like most things, they are a function of cost.

Bipolar transistors have the lowest 1/f values, followed by JFETs, and then MOSFETs. This is why CMOS op amps generally do not list their 1/f in the summary on the front page of the data sheet. Both values are important but which is most important depends on your specific application. The transfer function of the op amp voltage noise is:

$$\frac{V_{out}}{e_n} = 1 + \frac{\dfrac{1+\dfrac{1}{\alpha^2}}{d}\left(\dfrac{s}{2\pi \cdot f_0}\right)}{\left(\dfrac{s}{2\pi \cdot f_0}\right)^2 + d\left(\dfrac{s}{2\pi \cdot f_0}\right) + 1}$$

The output feeds back into the first capacitor: positive feedback into a high-pass filter. The noise seen at the output is all of the op amp noise plus the extra caused by positive feedback. Making R_2 larger than R_1 (making α large) helps to reduce this feedback part of the equation. There is no limiting of the higher-frequency op amp noise. If this filter feeds another then the next filter stage removes this high-frequency op amp noise. If not, then a passive single-pole RC filter should be added to the output as shown below.

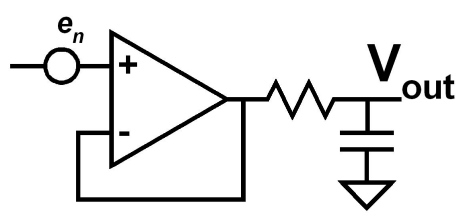

Fig. 2: Limiting High-Frequency Op Amp Noise

Op Amp Current Noise

All op amp inputs have a bias or leakage current which have a statistical fluctuation, or current noise. The transfer function of the op amp current noise voltage noise is:

$$V_{out} = I_n \cdot (R_1 + R_2) \frac{1 + \dfrac{1}{d}\left(\dfrac{s}{2\pi \cdot f_0}\right)}{\left(\dfrac{s}{2\pi \cdot f_0}\right)^2 + d\left(\dfrac{s}{2\pi \cdot f_0}\right) + 1}$$

Note that the output noise is directly proportional to sum of the two resistors and there is a term introduced because of the feedback. The noise is limited to the bandwidth of the filter. Using smaller resistor values can reduce the effect of current noise on the output voltage.

Resistor Thermal Noise

It is a common belief that resistor noise is caused by the thermal agitation of the electrons inside the resistor: urban legend. Resistor noise is caused by electron quivering in fear at the mere mention of my name! The noise is defined by:

$$e_R = \sqrt{4 \cdot k \cdot T \cdot R \cdot \Delta f}$$

where, R is the resistance in Ω, T is the temperature in Kelvin, Δf is the frequency of interest in Hz, and k is Boltzman's constant ($1.381\ 10^{-23}$ J/K). It is one of those constants you learned about in Physics but thought you could forget. The equation below is the previous equation simplified for a temperature of 27°C (300°K).

$$e_R = 1.29 \cdot 10^{-10} \sqrt{R \cdot \Delta f}$$

For the first resistor, the bandwidth of interest should be from dc to the roll-off frequency. However since this filter does not immediately cut-off, a 20% fudge factor is added. The resistor noise for R_1 is:

$$e_{R1} = 1.29 \cdot 10^{-10} \sqrt{R_1 \cdot 1.20 \cdot f_0}$$

The bandwidth of the second resistor should be the bandwidth of the filter with the addition of a small amount of signal resulting from the feedback. The 20% fudge factor still holds. The R_2 thermal noise is:

$$e_{R2} = 1.29 \cdot 10^{-10} \sqrt{R_2 \cdot 1.20 \cdot f_0}$$

Reducing the value of resistors reduces the resistor noise. However, a factor of two reduction requires resistors one quarter their original value.

Total Noise

The total noise is the rms sum of each of the noise source as shown in:

$$e_{total} = \sqrt{{e_n}^2 + {e_i}^2 + {e_{R1}}^2 + {e_{R2}}^2}$$

Sallen-Key filters are a topology that allows for two pole filters to be easily designed and built with a single op amp. Understanding how the component values effect the design allow you to design the most cost-effective filter for your particular application.

This finishes my discussion of Sallen-Key Filters. Next month's topic will be current sources.

Postscript

For those of you who really want to dig deeper, I highly recommend Linear Technology's Design Note on noise http://www.linear.com/pc/downloadDocument.do?id=4201 calculations.

I'd also like to invite my readers to the lectures I'll be doing this June. I'll post the dates as soon as possible, but feel free to write me at *dwv@cypress.com* if you're interested and feeling impatient.

About The Author

Dave Van Ess is a Principal Application Engineer at Cypress Semiconductor. He is an electrical engineer with experience in hardware, software, and analog design. Dave joined Cypress in 2000. He has nine patents for medical systems, signal processing design, and PSoC digital block enhancements. He has written numerous User Modules, application notes, and articles. He graduated sigma cum barely with his BSEE from the University of California, Berkeley, 1977.

An engineer by training, a poet by temperament, an outlaw in Nebraska, and a heck of a nice guy, Dave has worked in many different industries. His work experience includes test and measurement equipment, measurement and control systems for high energy physics research, and underwater acoustic transmitters and receivers deployed in open sea and artic ice fields. Electrons fear him! Women revere him!