Signals-From-Noise

What Sallen-Key Filter Articles Don't Tell You

Part I: What are Sallen-Key filters, what makes them tick, and what makes them so tricky to use?

by Dave Van Ess, Principal Application Engineer, Cypress Semiconductor

It seems that six months can't go by without some article in an electronic design magazine showing how to select components for a Sallen Key filter. Articles written before the 70s generally relied on nomograms to select component values http://thesaurus.maths.org/mmkb/entry.html?action=entryById&id=1495 or maybe a FORTRAN program that ran on some big mainframe computer. In the late 70s, articles started showing up with programs written for the venerable Radio Shack *Trash 80* http://skylane.kjsl.com/trs80/. The advent of the IBM personal computer led to the programs being written in either visual basic or C. Articles now are most likely programs written in JavaScript. (*Acquire some electronic magazines of the 80s vintage, find an article to paraphrase, rewrite the program into JavaScript and in no time at all you are a published engineer.*) These articles make designing a filter seem trivial. What are left out are the real world constraints that keep the designer from getting the best performance. Below are the answers to the questions other articles don't explain.

Who are Sallen and Key and what makes them so special?

R P Sallen and E L Key were engineers at Lincoln Labs, MIT. They published*, "A Practical Method of Designing Active Filters," *IRE Transactions on Circuit Theory,* vol CT-2, pp 74 to 85, March 1955. It was an amazing paper that showed that high quantity filters could be made without the use of inductors. This paper defined all the basic topologies, or networks as they called them, used today. Although the examples were done with triodes, they can easily be translated to op amp circuits. I believe it to be one of the must-read papers for electrical engineers. Now for the quiz:

1. Who was the more senior of these two engineers?
2. Who did most of the tedious work?

When I was a young engineer, this type of arrangement did not impress me. Now that I am older, wiser, and have a better understanding of worldly matters, I understand the wisdom of such a system.

Low-Pass Filter Basics

A low-pass filter passes frequencies below some defined roll off frequency (f_0) and rejects frequencies above it. A transfer equation for a unity gain second-order low pass filter is defined as:

$$\frac{V_{out}}{V_{in}} = \frac{1}{\left(\frac{s}{2\pi f_0}\right)^2 + d\left(\frac{s}{2\pi f_0}\right) + 1}$$

The only two variables are the roll-off frequency (**f_0**) and the damping value (**d**). The roll-off frequency is the point where the "s" terms start to dominate. Frequencies below this value are considered "low" and above this value are considered "high." Damping determines how the filter transitions from lower to higher frequencies. At f_0, the transfer equation reduces to 1/d. These filter stages can be cascaded to produce higher-order filters. All that is required is the roll-off frequencies and damping values for the particular filter you desire.

Fortunately, these can be found in most any filter design book and will not require any complex calculations. I recommend the *Active Filter Cookbook* by Don Lancaster http://www.amazon.com/Active-Filter-Cookbook-Donald-E-Lancaster/dp/0672211688. (I seem to always be buying a new copy because mine keeps disappearing. I can think of no higher compliment for a book than to say it is steal-able.) Nowhere in this column is there any reference to poles and zeros. They are not required to design a filter. So, as far as you need to be concerned, poles are places that are really cold and zeros are the folks you don't want your kids hanging out with.

The Basic Sallen-Key Low-Pass Filter

The topology for a second-order Sallen-Key, unity-gain, low-pass filter is shown below.

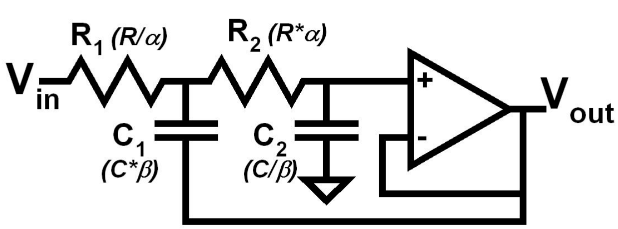

Fig. 1: Basic Sallen-Key Low-Pass Filter Topology

$$\frac{V_{out}}{V_{in}} = \frac{1}{(sRC)^2 + \frac{\alpha + \frac{1}{\alpha}}{\beta}(sRC) + 1}$$

Combining the two previous transfer equations allows the roll-off frequency and damping value to be determined:

$$f_0 = \frac{1}{2\pi \cdot R \cdot C} \quad R = \sqrt{R_1 R_2} \quad C = \sqrt{C_1 C_2}$$

$$d = \frac{\alpha + \frac{1}{\alpha}}{\beta} \quad \alpha = \sqrt{\frac{R_2}{R_1}} \quad \beta = \sqrt{\frac{C_1}{C_2}}$$

Implementing A Filter

Are you tired of your company's heavy-duty analog muscle kicking sand in your face? Well in just seven steps I can make you a filter!

1 Determine the desired roll-off frequency and damping value. Suppose this filter is to be a second-order 1 kHz Bessel filter. Don's book shows a required roll off frequency of **1.272 kHz** and damping value of **1.732**

2 Select the capacitor values such that

$$\beta = \sqrt{\frac{C_1}{C_2}} \geq \frac{2}{d}$$

For this example $\mathbf{C_1}$ is set to **2000 pF** while $\mathbf{C_2}$ is set to **1000 pF**. $\boldsymbol{\beta}$ works out to be **1.414** while the mean capacitance is **1414 pF**

3 Determine the mean resistance:

$$R = \frac{1}{2\pi f_0 C} \quad \frac{1}{2\pi \cdot 1.272\ kHz \cdot 1414pF} = 88.49\ kOhms$$

4 Given $\boldsymbol{\beta}$ and the damping value (**d**) calculate $\boldsymbol{\alpha}$ using:

$$\alpha = \frac{d \cdot \beta + \sqrt{(d \cdot \beta)^2 - 4}}{2} \quad \frac{1.732 \cdot 1.414 + \sqrt{(1.732 \cdot 1.414)^2 - 4}}{2} = 1.931$$

5 Given $\boldsymbol{\alpha}$ and the mean resistance **R**, determine $\mathbf{R_1}$ and $\mathbf{R_2}$ with:

$$R_1 = \frac{R}{\alpha} \quad \frac{88.49kOhms}{1.931} = 45.83kOhms$$

$$R_2 = R \cdot \alpha \quad 88.49kOhms \cdot 1.931 = 170.9kOhms$$

6 Consult a 1% resistor value table to determine closest commercially-available resistor values

$$R_1 = 46.4kOhms$$

$$R_2 = 169kOhms$$

7 Use the selected values to calculate the actual roll-off frequency and damping value:

$$f_0 = \frac{1}{2\pi \cdot R \cdot C} = 1.271kHz \quad R = 88.55k \quad C = 1414.2pf$$

$$d = \frac{\alpha + \frac{1}{\alpha}}{\beta} = 1.7203 \quad \alpha = \sqrt{\frac{169\,k}{46.4k}} = 1.9085 \quad \beta = \sqrt{\frac{2000\ pf}{1000\ pF_2}} = 1.414$$

For this example the actual roll-off frequency is within 0.1% of the desired value and the actual damping value is within 1%.

Is It Really This Easy?

No! There are many solutions available. I could have just as easily selected resistors a decade bigger and capacitors a decade smaller, two decades bigger and smaller, etc. This particular design assumed an ideal op amp. Real op amp constraints will affect component selection. Here are some of the rules:

- It is best to make the resistors as large as possible to reduce the size of the capacitors
- It is best to make the resistors as small as possible to reduce the offset voltage generated by the op amp's input bias current
- It is best to make the resistors as large a possible to reduce the loading on the op amp's output
- It is best to make the resistor as small as possible to reduce their thermal noise

These rules just keep going but basically they are a bunch of conflicting constraints. Each rule will depend on what particular parameters are important for your specific application.

Capacitors

The range of acceptable capacitors limits the selectable resistor values. The capacitor of choice is the NPO ceramic capacitor. It is a special cut of the ceramic that has extremely good temperature stability allowing for tolerances down to ±0.5%. (Down to ±5% they stay moderately cheap.) You should avoid capacitor values below 100 pF as their stray node capacitance starts to have a relatively larger affect on its actual value. The upper limit for NBO capacitors is 0.1 µF. (At 0.01 µF they start to get pricey.)

Another option is the X7R ceramic capacitor. It is a different cut of the ceramic that allows for more capacitance in a given size. Unfortunately, it also has a larger variation over temperature. Practically speaking its best tolerance is ±10% but comes in values up to 10 µF. A good rule of thumb is to use NPO when you can and X7R when you have to.

Op Amps

The two primary dc op amp parameters are; input bias current, input offset voltage.

Input Bias Current

This is the amount of current that leaks from the op amp's input. It can be as low as a few pA for CMOS op amps, up to µA for a high-speed bipolar video op amp. The offset is defined by the equation:

$$V_{CurrentOffset} = I_{bias}\left(R_1 + R_2\right).$$

Given the bias current for a particular op amp and a budget for acceptable offset voltage, it is possible to set a maximum value for the sum of the resistors. An OP37 bias current is roughly 40 nA. Suppose the acceptable current offset is 1 mV and the sum of the resistors is limited to 25 kΩ:

$$\frac{V_{CurrentOffset}}{I_{bias}} = \frac{1mV}{40nA} = 25kOhms \geq \left(R_1 + R_2\right)$$

Input Offset Voltage

Because the op amp is configured as a buffer the input offset voltage is directly applied to the filter output. It is just an exercise in selecting a part with acceptable offset.

One of the primary parameters affecting the cost of an op amp is its offset voltage. If low output impedance is not required then following circuit removes the op amp offset constraint.

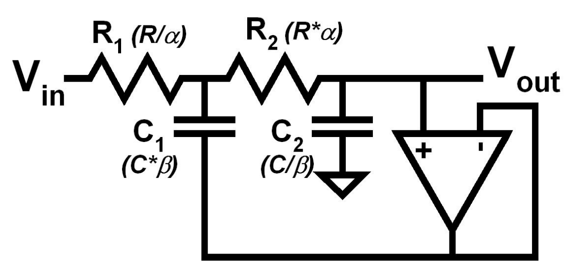

Fig. 2: Sallen-Key Low-Pass Filter With No Dc Feedback Path

Note that the op amp is not in the dc feedback path of the filter. Its input offset error has no effect on the filter output. With many ADCs having high impedance inputs the requirement for a low impedance output may not be necessary.

Now why didn't Sallen and Key figure this out? The short answer is that they went to MIT. If they had studied at CAL (Berkley) it would have been obvious to them. Collegial snobbery aside, back in the 1950s it was harder to generate a high input impedance and there would have been little application for such a filter.

Now that you have a feel for the basics of Sallen-Key filters, it's time to put them to work. In the next installment of *Signals from Noise*, we'll explore what effect the following parameters have on filter performance and component selection.

- Op amp gain bandwidth and output impedance
- Op amp voltage noise
- Op amp current noise
- Resistor thermal noise

E-mails with questions, comments, or suggestions for future installments of this column are all welcomed: write me at dwv@cypress.com

Postscript

*For those with a burning curiosity about the origins of these incredibly handy and often-misunderstood circuits, we would have liked to have put a link to a copy of the original paper. However, under the Sonny Bono Copyright Term Extension Act of 1998 (often called the Mickey Mouse Protection Act) the copyright on this corporately-written paper extends for 95 year from publication: March 2050. I will be happy to send you a PDF copy of the paper provided you agree you will make appropriate payment to one of the Copyright Clearing Services (ask at any library). The piece is so well worth reading, and so interesting, that I have long searched for an original copy of my own. In fact, I am willing to pay $100 for an original copy of this journal. Now I am not encouraging students to remove a copy in their engineering library. That would be stealing and stealing is bad.

About The Author

Dave Van Ess is a Principal Application Engineer at Cypress Semiconductor. He is an electrical engineer with experience in hardware, software, and analog design. Dave joined Cypress in 2000. He has nine patents for medical systems, signal processing design, and PSoC digital block enhancements. He has written numerous User Modules, application notes, and articles. He graduated sigma cum barely with his BSEE from the University of California, Berkeley, 1977.

An engineer by training, a poet by temperament, an outlaw in Nebraska, and a heck of a nice guy, Dave has worked in many different industries. His work experience includes test and measurement equipment, measurement and control systems for high energy physics research, and underwater acoustic transmitters and receivers deployed in open sea and artic ice fields. Electrons fear him! Women revere him!

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