Circuit Design Clinic
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Mixed-Technology Laser System, Part 4
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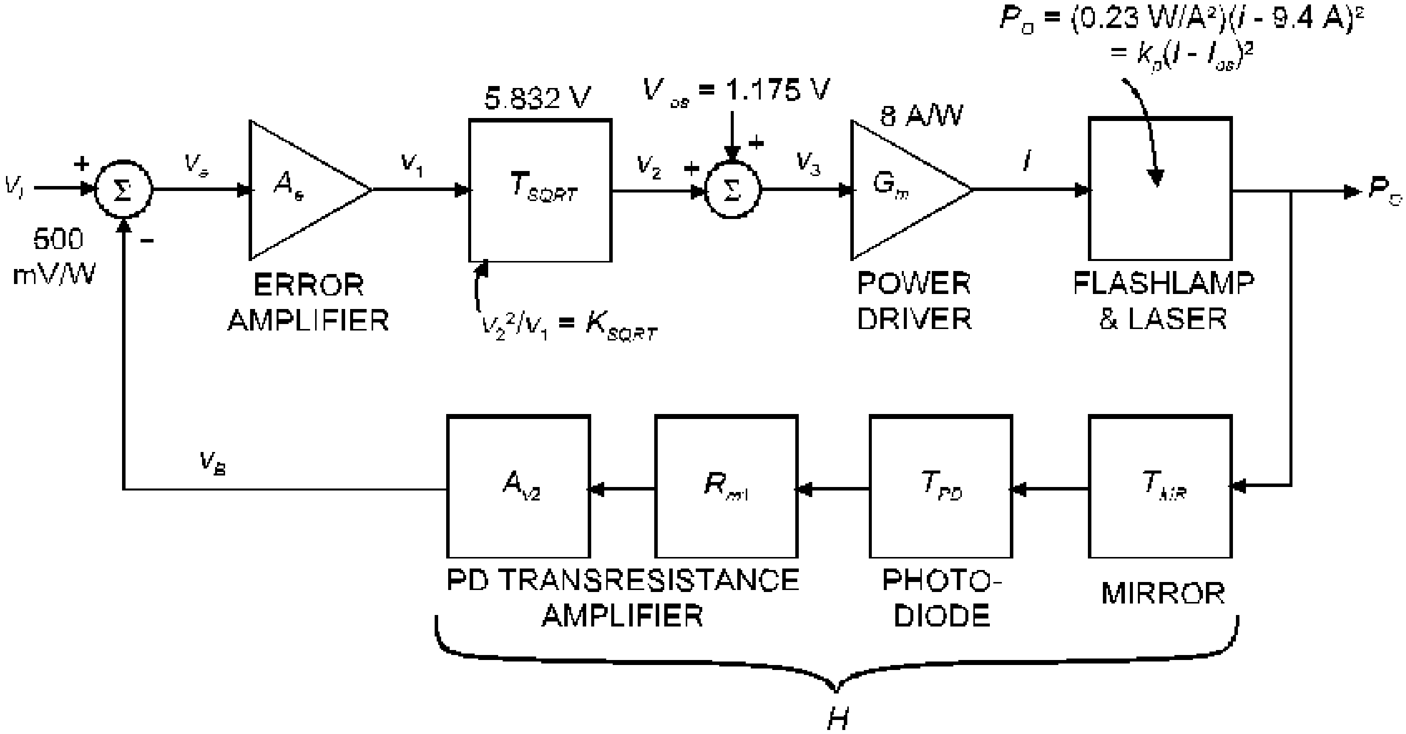
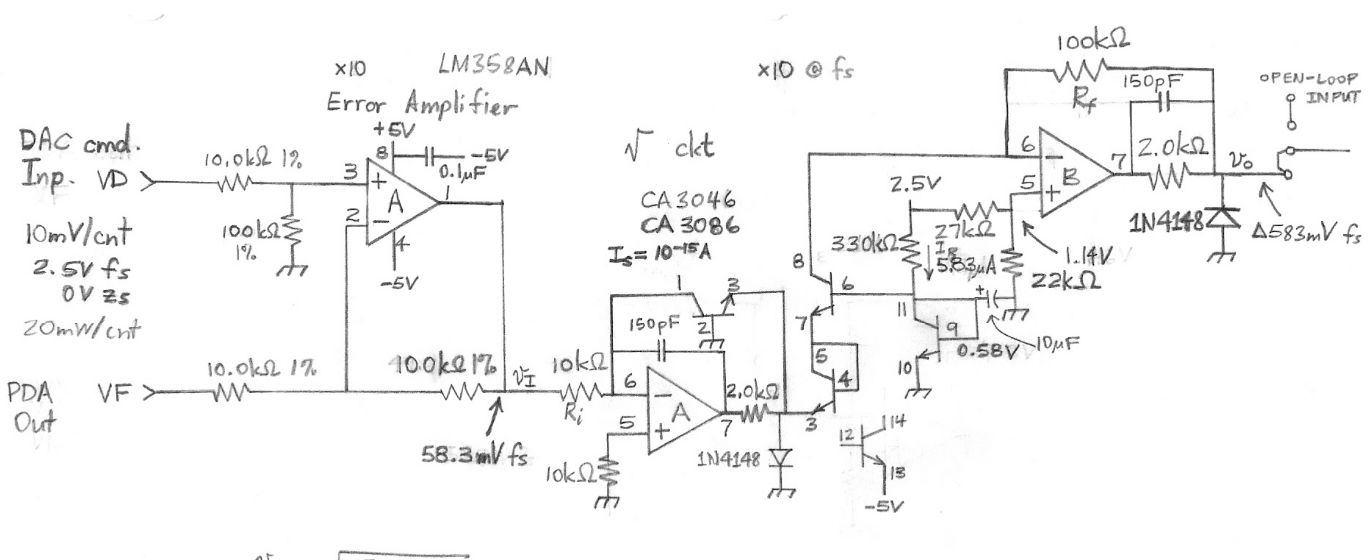
The previous article in this series began investigation of the problem of Nd YAG laser power control for a 5 W fs output. In this fourth part, the design of the whole laser loop is completed, beginning with the dynamics. The loop block diagram is repeated from the previous part, below.



Dynamic Analysis

Now we will proceed with dynamic analysis, one block at a time, until we have traversed the loop, then put it all together into a loop-gain frequency-response plot and predict behavior. The G_1 circuitry is repeated below, from the project notebook.



$$\frac{v_o}{\sqrt{v_I}} = \sqrt{\left(\frac{R_f}{R_i}\right) R_f I_R} = 2.415\ V^{1/2}$$

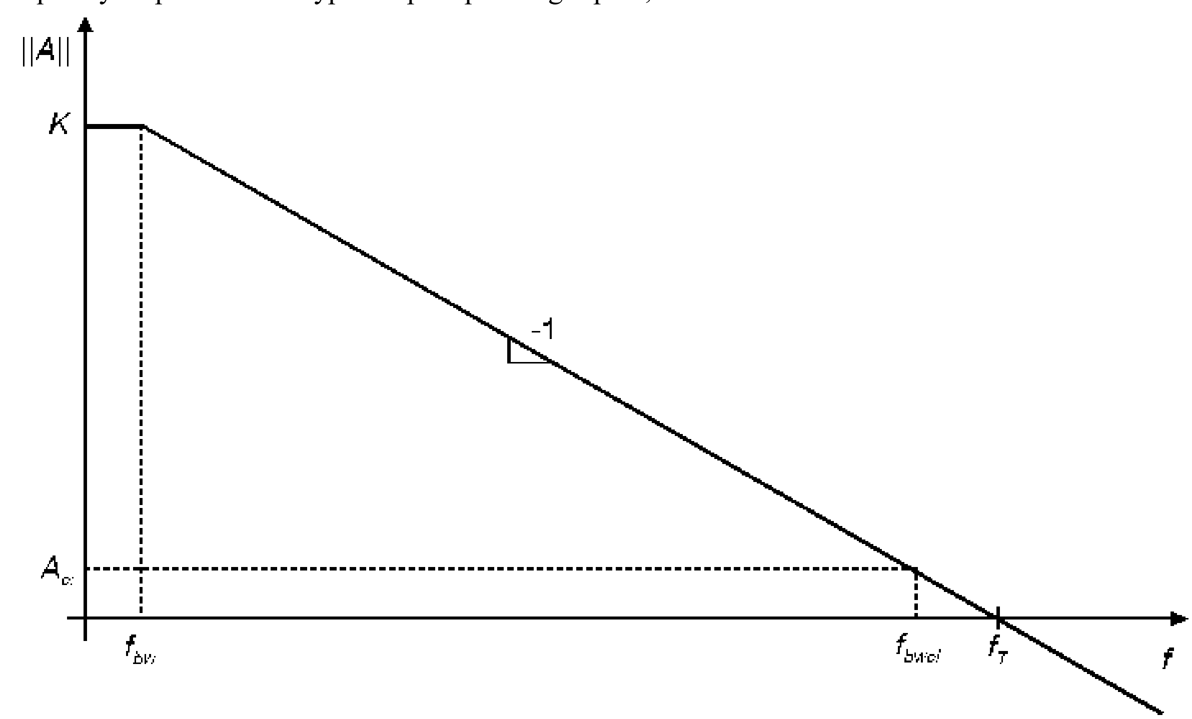
Error Amplifier

The error amplifier is a one-op-amp differential amplifier using a LM358, which has a GBW = f_T= 1 MHz. The transfer function of this inherently linear block is:

$$A_e = \frac{v_1}{v_e} = \frac{v_1}{v_i - v_B} = A_{e0} \cdot \frac{1}{s(\tau_T / A_{e0}) + 1}$$

where, v_i is the commanded output power, and v_B the *H*-path feedback out of the photodiode (PD) amplifier (PDA). When using low-cost (and low-speed) op amps, it is generally good practice to include the speed of the op amp in the feedback equations. This at first appears to be a complicating nuisance leading to unwieldy algebra, but it is not that hard to include.

The frequency response of the typical op amp is single-pole, as shown below.



This is the asymptotic approximation to the magnitude of the gain, on a log-log plot. The frequency response decreases from op-amp open-loop bandwidth at f_{bw} with a –1 slope, from a quasistatic, open-loop voltage gain of K, until it intersects the unity-gain axis at f_T. Then with a single-pole rolloff of –1:

$$f_T = K \cdot f_{bw}$$

For a typical $K = 10^5$ and $f_T = 1$ MHz, then the open-loop op-amp bandwidth is quite low; $f_{bw} = 10$ Hz.

The quasistatic closed-loop gain value, $A_{cl} \ll K$, and the resulting bandwidth, f_{bwcl} is proportionately higher; in the case of $A_e(f)$:

$$f_{bwcl} = \frac{f_T}{A_{cl}} = \frac{1\,\text{MHz}}{10} = 100\,\text{kHz}$$

Consequently, the error amplifier has contributed a single pole at 100 kHz and no zeros.

Now here is the simplifying technique for including op amp speed. Because K is so large relative to A_{cl}, for dynamic analysis, we can let K go to infinity. When this occurs, the op-amp transfer function:

$$A(s) = \left.\frac{K}{s\tau_{bw} + 1}\right|_{K \to \infty} = \frac{1}{s(\tau_{bw} / K)} = \frac{1}{s\tau_T}$$

Whenever an op amp feedback circuit is analyzed, use $1/s \cdot \tau_T$ instead of K to obtain the infinite-K approximation of the dynamic response, where:

$$\tau_T = 1/\omega_T = 1/2\pi f_T$$

For an op amp with $f_T = 1$ MHz, t_T @ 159 ns.

Square-Root Circuit

The input stage of the SQRT circuit is the op amp log stage, with its BJT from input to output. It also has a capacitor from op amp output to inverting input. The dynamic analysis of this circuit produces the following approximate transfer function:

$$A_v \cong -\frac{r_e / \alpha}{R_i} \cdot \frac{1}{s\left(\frac{R_o + r_e}{\alpha}\right) \cdot C_f + 1}$$

The approximation assumes the op amp pole is well beyond (around a decade higher than) the one given above.

The problem with this circuit is that the 50-to-1 dynamic range corresponds with a 50-to-1 range in input and BJT current. Consequently, r_e also varies by 50-to-1. In retrospect, the current range should be higher. It is 5.83 mA fs and a measly 117 nA zs. Although the circuit works in the application, I would now change R_i from 10 kΩ to 1.0 kΩ (or less) and R_f from 100 kΩ to 10 kΩ or less, thus boosting both v_1 (from 58.3 mV to 583 mV fs) and SQRT input current by a decade. Perhaps then, $C_f = 150$ pF and $R_o = 2.0$ kΩ could also be eliminated.

The wide variation in r_e causes the quasistatic gain and pole location to vary, from –22.3 and 4.71 kHz at zs to a much-reduced –0.65 and 163 kHz at fs. Two loop analyses will be needed for the two ends of the range. The values of r_e are: 221 kΩ zs and 4.43 kΩ fs.

The SQRT translinear BJTs are fast, with f_Ts of 550 MHz, and circuit poles well into multiple MHz, beyond the need for consideration. The squaring stack of two BJTs forms a CB stage, which is inherently fast. The output current is applied to the output op-amp, with $R_f = 100$ kΩ to develop the output voltage. The noninverting input is also offset by V_{os} (or as close as 5% resistors will allow -- another place to refine the design, with 1% parts).

The output-stage op-amp is also part of an LM358. Its transfer function, using an inverting op-amp $G = -1/s\cdot\tau_T$, is:

$$Z_m(s) = \frac{v_2}{i_1} = T_i \cdot \frac{G}{1+GH} = (R_f)\cdot\left(\frac{-1/s\tau_T}{1+(-1/s\tau_T)\cdot(-1)}\right) = -R_f \cdot \frac{1}{s\tau_T + 1}$$

Consequently, the SQRT output stage has a pole at the op-amp f_T of around 1 MHz. We will find later that this is of some significance.

Flashlamp Driver

The laser flashlamp current driver was a purchased motor drive, adapted to the application. It outputs over 13 A. I did not want to modify it, causing this purchased subsystem to become customized, because any changes made within it by its supplier were outside our control and would force us to modify our own customization of it. The schematic diagram of it was available, but its complexity did not encourage the kind of analysis of it that we are doing here of the laser loop. Its behavior was investigated on the bench. Measurements produced an approximation of its frequency response, with a pole at 1.8 kHz and a zero at 7.1 kHz. The manufacturer's specification gave a 2 kHz bandwidth.

The flashlamp-laser subsystem dynamics were unknown, but are believed to be relatively fast. The bench measurements of the driver included flashlamp loading. The assumption for this analysis is that no appreciable affect is contributed by this subsystem to the loop response.

PDA

The photodiode amplifier has four stages, including the laser-beam pickoff mirror. The simplified PDA model is shown below. The mirror is essentially an optical power divider. Its transfer function or *transmittance* is:

$$T_{MR} = \frac{i_i}{T_{PD}\cdot P_O} = \frac{15\ \mu\text{A}}{(0.65\ \text{A/W})\cdot(5\ \text{W})} = \frac{15\ \mu\text{A}}{3.25\ \text{V}} = 4.62\cdot 10^{-6} = 4.62\ \text{ppm}$$

The mirror passes only a tiny fraction of the total incident laser beam. Very small fractions can be potential trouble indicators, yet except for the problem that the divider ratio is different depending upon light polarization, it has worked acceptably in practice as an optical divider.

The overall transresistance of the PDA is:

$$R_m = \frac{v_o}{i_i} = \frac{2.5\ \text{V}}{15\ \mu\text{A}} = 166.7\ \text{k}\Omega$$

The first amplifier stage transresistance:

$$R_{m1} = R_f = 10\ \text{k}\Omega$$

The output voltage amplifier stage has a gain of:

$$A_{v2} = \frac{R_m}{R_{m1}} = \frac{166.7\ \text{k}\Omega}{10\ \text{k}\Omega} = 16.67 = \frac{R_f}{R_i} + 1$$

Then

$$R_i = \frac{R_f}{A_{v2} - 1} = \frac{20\ \text{k}\Omega}{15.67} = 1.3\ \text{k}\Omega$$

A magnitude frequency-response plot for the PDA shows a dominant pole rolling off at 106 Hz. At 150 kHz, another pole is encountered and the pole slope changes from –1 to –2. It encounters a zero at 796 kHz, changing the slope back to –1, but not for long. At 813 kHz, another pole changes the slope to –2, near the unity-gain crossover frequency, f_c. Way out at 14.88 MHz a final pole within sight bends the slope to a precipitous –3, but well beyond f_c, where any harm can be done.

Loop Analysis

Now comes the moment for putting all the poles and zeros from the loop blocks together into a Bode plot. We will do this twice, at each end of the operating range, because of that shifting pole between zs and fs. The following table, in ascending frequency, is what we must plot. Keep in mind that the quasistatic loop gain, $G_0 \cdot H_0$ =429.

Poles	**Zeros**	**Subsystem**
106 Hz		Dominant pole; PDA LPF
1.8 kHz		G_m amplifier
4.71 kHz zs		SQRT circuit log amplifier
	7.1 kHz	G_m amplifier
100 kHz		Error amplifier
150 kHz		PDA A_{v2}
163 kHz fs		SQRT circuit log amplifier
	796 kHz	PDA A_{v2}
813 kHz		PDA R_{m1}
1 MHz		SQRT circuit
14.9 MHz		PDA A_{v2}

The full-scale response is plotted below. The interesting part occurs around f_c, where it crosses unity gain. Generally, if the slope around f_c is too negative, excess phase will have accumulated, causing the phase margin, a measure of stability, to be too small.

We will use asymptotic linear approximations for phase. A pole or zero will affect phase a decade each side of it, and will shift phase from a decade below to a decade above by Δ90°, –90° for a pole and +90° for a zero. The linearized slope of phase for a pole or zero has a magnitude of 45°/decade. Linear interpolation of frequencies on a log-log plot produces the phase contribution at a given frequency. In general, if a pole is at f_p, then the phase contribution of this pole at frequency f is:

$$\Delta\phi = -45° \cdot \log_{10}\left(\frac{f}{f_p/10}\right)$$

At $f_p/10$, there is no significant phase lag. (Actually, it is about –6°.) At the pole frequency, f_p, it is –45°. And at $10 \cdot f_p$, the phase contribution is a full –90°. It works the same with zeros, except their phase contribution is positive. Hence we use zeros to cancel the phase lags of poles, to keep the phase from descending to near –180° before f_c is reached. The indicator of how much stability the loop has is the amount that the phase is above –180° at f_c, which is the *phase margin*. It is the indicator of dynamic behavior being sought.

Besides computing phase accumulation with f, we must also plot the magnitude of GH so that we can determine f_c. As the magnitude slope, m, changes, the decrease in loop gain from frequency f_1 to f_2 is:

$$\left(\frac{A_2}{A_1}\right) = \left(\frac{f_2}{f_1}\right)^m$$

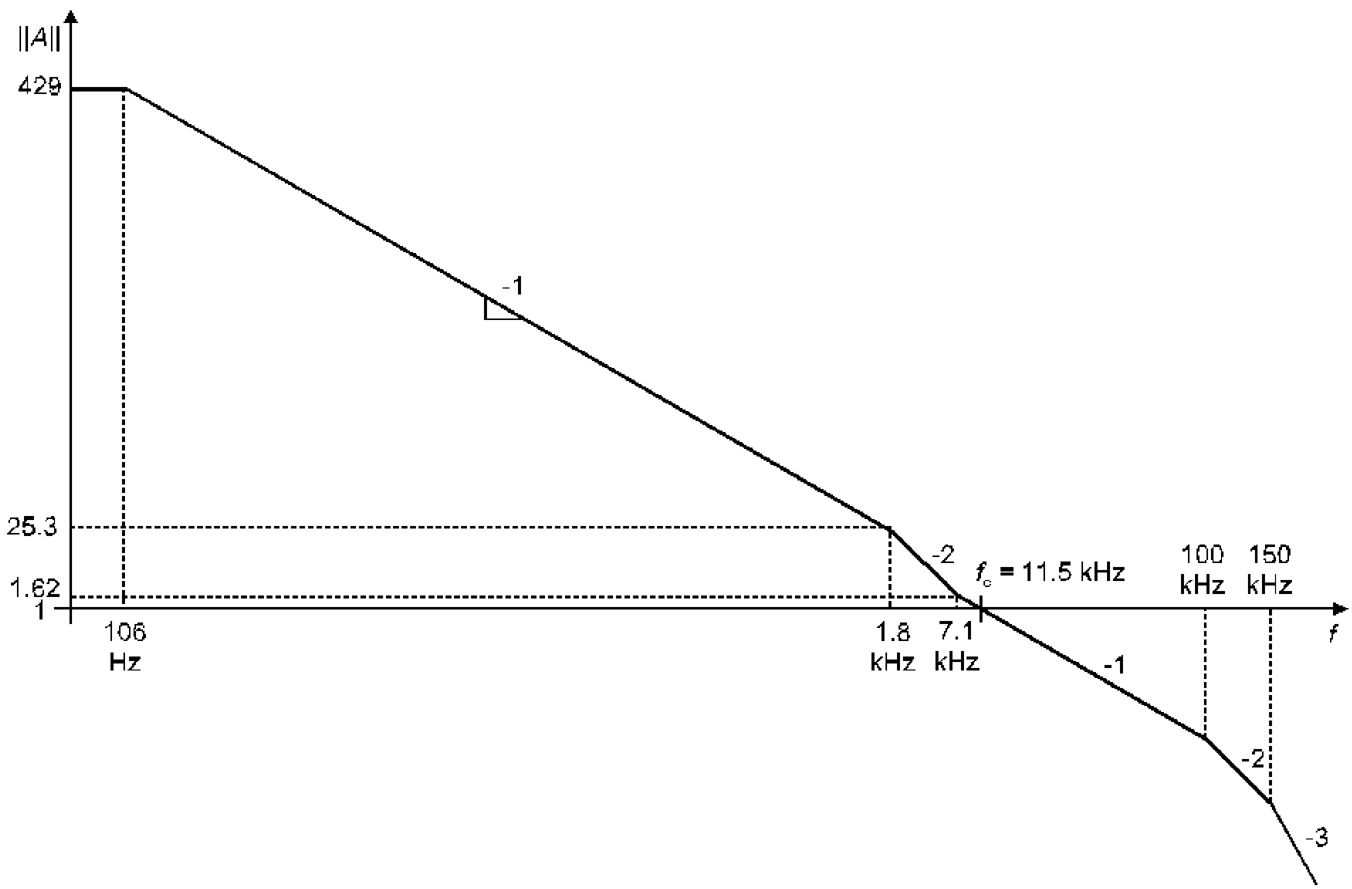
For the laser loop, the gain is 429 at the first pole, at 106 Hz. The second pole occurs at 1.8 kHz. What is the gain at the frequency of the second pole? Apply the formula:

$$\left(\frac{A_2}{429}\right) = \left(\frac{1.8\ \text{kHz}}{106\ \ \text{Hz}}\right)^{-1}$$

Then:

$$A_2 = 429\cdot\left(\frac{1.8\,\text{kHz}}{106\,\text{Hz}}\right)^{-1} = 429\cdot(0.0589) = 25.26$$

This can be seen marked on the fs plot.



Now that the magnitude plot has been calculated, the phase is next. It is –45° at the first pole and being more than a decade below f_c, contributes a full –90° at f_c. The 1.8 kHz pole contributes:

$$\Delta\phi = -45°\cdot\log\left(\frac{11.5\,\text{kHz}}{1.8\,\text{kHz}/10}\right) = -81.2°$$

The 7.1 kHz zero contributes:

$$\Delta\phi = +45°\cdot\log\left(\frac{11.5\,\text{kHz}}{7.1\,\text{kHz}/10}\right) = +54.4°$$

The 100 kHz pole contributes –2.7°. The poles at higher frequencies can be considered out of range to have any effect. Accumulating the phase contributions from the contributing poles and zeros, the result is –120°, and the phase margin, *PM*, is –120° –(–180°) = 60°. Is this sufficient?

A two-pole feedback system with a phase margin between zero and 64° will have a closed-loop damping ratio:

$$\zeta_c \cong \frac{PM}{100^\circ}$$

and a closed-loop fractional overshoot to a step input, for $PM > 20°$, of:

$$M_{pc} = 75 - PM,\ M_{pc} \text{ in \%, } PM \text{ in deg, } PM > 20°\text{, 2nd-order}$$

The closed-loop pole angle is thus:

$$\phi = \cos^{-1}\zeta_c$$

For our case, the closed-loop pole angle is around 53°, an angle in which overshoot of a step input will ring through little more than one cycle before being damped. The overshoot, which might be more of a concern since it involves peak power, is 15%. While the laser loop is not a two-pole loop, the simpler two-pole case at least gives us a clue as to what the response might be.

The zs case is more of a stability concern because of the additional 4.71 kHz pole due to the log amp of the SQRT circuit. At low current, the pole moved from its 163 kHz fs value to well within the frequency range of the loop dynamics. When the zs frequency-response analysis is carried out, f_c is found to be 7.38 kHz. The log-amp pole causes a brief –3 slope near f_c -- definitely a bad sign -- but is quickly corrected by the zero at 7.1 kHz. The proximity in frequency of the zero to the pole essentially causes the zero to cancel it, not allowing it to add much phase lag to an already lagging loop. When the phase at f_c is calculated, it is not much worse than the fs case: –126°, and a $PM = 54°$.

Although this loop could be better optimized for dynamic response, beginning perhaps with a new, improved log-amp that doesn't shift its pole with input current, it functions acceptably in the application with the above modifications to the original, essentially hopeless, loop. The key was the SQRT linearization.

The lesson in this design applies as a general principle: linearize open-loop blocks first before depending upon feedback to do the rest. This applies as much to laser power controllers as to low-distortion audio amplifiers.