Wire Packing Factor in Magnetics Design

by Dennis L Feucht

Packing factor is a quantity that appears in various places in electronics. For example, the distribution of pixels on the faceplate of a video display are made as dense as possible, to increase both resolution and active light-sourcing area. The packing factor is the fraction of a total area that is used as desired. Ideally, packing factor, $k_p = 1$.

In magnetics design, transductors (transformers or inductors) usually contain windings of round wire in rectangular winding windows, of area A_w. To calculate the cross-section of total conductor area within the window, it is necessary to know k_p, for then the total conductive area for N turns of wire is:

$$N \cdot A_c = k_p \cdot A_w$$

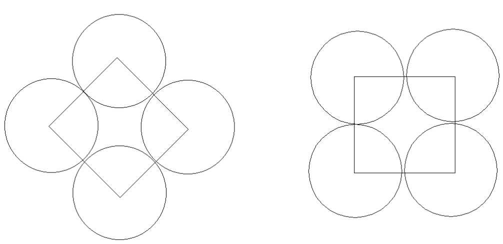
where, A_c is the single-wire conductive area or wire-gage area. An analytic formula for k_p has been difficult to find. One now-defunct magnetic core manufacturer, Indiana General, once published in their catalog an empirical curve of k_p as a function of wire size, as measured in the American Wire Gage (AWG). In writing a MathCAD program to automate magnetics design, I was faced with deriving $k_p(A_c)$.

Before presenting this derivation, I recently discovered (while reading *Scientific American*, FEB98, pp.94 - 95, 97, *Tight Tins for Round Sardines*, Mathematical Recreations, by Ian Stewart) that there is a branch of mathematics called *combinatorial geometry* that addresses the question of how to maximally pack, for example, n circles into a given square. It is essentially k_p math.

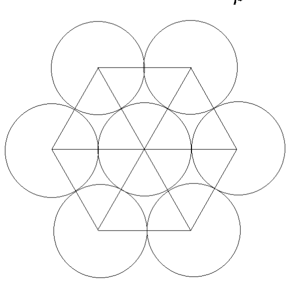
k_p Derivaton

I began my investigation of how to derive k_p by thinking about how rows of wires lay down next to each other, without regard to window constraints. It became evident that a suboptimal packing factor is obtained when the centers of the wires are positioned at the vertices of squares, as shown below. The best packing resulted in a hexagonal pattern.

Minimum k_p



Maximum k_p



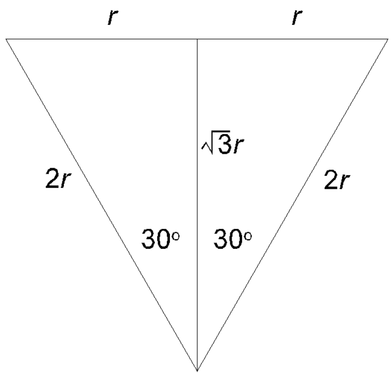
Wire radius = r
Conductor radius = r_c
Minimum k_p configurations:
Square and diamond packing have same k_p - rotated 45°
$A_{sqr} = (2 \cdot r)^2 = 4 \cdot r^2$
Area of wire within square =

$$A_{wsqr} = 4 \cdot \left(\frac{1}{4} \cdot \pi \cdot r^2\right) = \pi \cdot r^2$$

$$k_{pwsqr} = \frac{A_{wsqr}}{A_{sqr}} = \frac{\pi}{4} \cong 0.785$$

Hexagonal packing: 6 equilateral triangles with area,

$$A_{tri} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot (2 \cdot r) \cdot (\sqrt{3} \cdot r)$$
$$= \sqrt{3} \cdot r^2$$



Each circular sector within hexagon has area,

$$A_{\sec t} = \frac{\theta}{2} \cdot r^2 = \frac{\pi}{6} \cdot r^2$$

$$k_{pwhex} = \frac{3 \cdot A_{\sec t}}{A_{tri}} = \frac{3 \cdot \left(\frac{\pi}{6}\right) \cdot r^2}{\sqrt{3} \cdot r^2} = \frac{\pi}{2 \cdot \sqrt{3}} \cong 0.907$$

where k_{pw} = *wire* packing factor. For conductor only,

$$k_p = \left(\frac{r_c}{r}\right)^2 \cdot k_{pw}$$

where, for heavy (double) insulated wire:

$$\left(\frac{r_c}{r}\right)^2 = \frac{r_c}{r_c + \sqrt{a \cdot r_c}} = \frac{1}{1 + \sqrt{\frac{a}{r_c}}}, \; a \cong 392 \cdot 10^{-6} \text{ cm}; r_c = (0.41264 \text{ cm}) \cdot (1.123)^{-\text{AWG}}$$

$r - r_c$ = insulation thickness

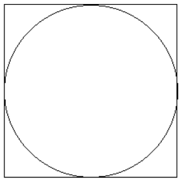
The packing factor range is between minimum and maximum:

$$k_p \in \left\{ \left(\frac{r_c}{r}\right)^2 \cdot \frac{\pi}{2 \cdot \sqrt{3}}, \left(\frac{r_c}{r}\right)^2 \cdot \left(\frac{\pi}{4}\right) \right\}$$

For a random wind, these patterns will be mixed. I took the average as midway between extremes; Average packing factor =

$$\bar{k}_p = \left(\frac{r_c}{r}\right)^2 \cdot \underbrace{\left(\frac{(\sqrt{3}+2) \cdot \pi}{8 \cdot \sqrt{3}}\right)}_{0.846}$$

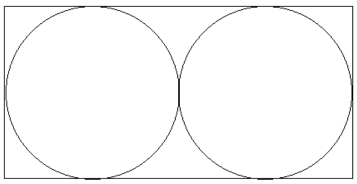
Then I brought in the rectangular window constraint on k_p, beginning with a single wire in a square window, followed by a single row of wire, and then multiple rows.



- Winding window affects k_p:

Ends of window constrain wire:

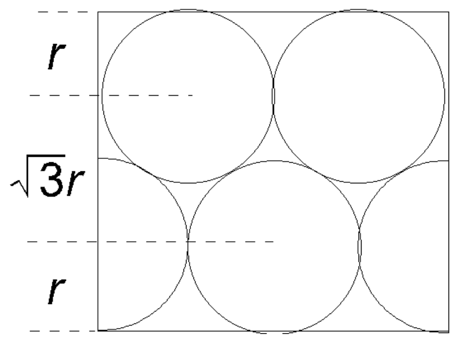
$$k_p = \frac{\pi \cdot r^2}{(2 \cdot r)^2} = \frac{\pi}{4} \cong 0.785$$



$$k_p = \frac{2 \cdot \pi \cdot r^2}{(2 \cdot r) \cdot (4 \cdot r)} = \frac{\pi}{4} \cong 0.785$$

Hex configuration (max k_p), rectangular window: m wires wide $\times$ n layers

Area of window =



$$A_{wndw} = (2 \cdot m \cdot r) \cdot [(2 + \sqrt{3} \cdot (n-1)) \cdot r]$$

Total wire area within window =

$$A_{wire} = \begin{cases} m \cdot n \cdot \pi \cdot r^2, & \text{full occupancy : no missing center wires} \\ \left(\frac{3}{2} \cdot m\right) \cdot \left(\frac{n}{2}\right) \cdot \pi \cdot r^2, & \text{missing center wires from hex configuration} \end{cases}$$

Then wire packing factor is

$$k_{pw} = \frac{A_{wire}}{A_{wndw}} = \begin{cases} \dfrac{\left(\frac{n}{2}\right) \cdot \pi}{2 + \sqrt{3} \cdot (n-1)}, & \text{full occupancy} \\ \dfrac{3}{4} \cdot \dfrac{\left(\frac{n}{2}\right) \cdot \pi}{2 + \sqrt{3} \cdot (n-1)}, & \text{missing centers} \end{cases}$$

For layers $n \to \infty$:

$$k_{pw}(n \to \infty) = \begin{cases} \dfrac{\pi}{2 \cdot \sqrt{3}} \cong 0.907 \\ \dfrac{3}{4} \cdot \dfrac{\pi}{2 \cdot \sqrt{3}} \cong 0.680 \end{cases}$$

Average k_{pw} =

$$\bar{k}_{pw} = \frac{7}{8} \cdot \frac{\left(\frac{n}{2}\right) \cdot \pi}{2 + \sqrt{3} \cdot (n-1)} \xrightarrow{n \to \infty} \frac{7}{8} \cdot \frac{\pi}{2 \cdot \sqrt{3}} \cong 0.765$$

With this expression for an average k_p, I wrote a MathCAD program to produce a graph that can be used for magnetics design. This program can also be incorporated into a larger program (such as the Innovatia http://www.innovatia.com Magnetics Design designware package).The following page can be printed and used for k_p estimation for magnetics design.

Closure

How well does the derived k_p conform to reality? I have used it in magnetic design several times and it tends to be slightly low, but not by much. (Some of this might be caused by assuming triple insulation for magnet wire when usually double (heavy) insulation is used.) This is usually the preferred direction of error, for if the winding window is not completely full, the device can still be manufactured. But if k_p errs high (optimistic), the desired number of turns might not be able to fit in the window and the device cannot be built as specified.

The function shows that as wire size decreases, so does packing factor. This graph does not take into account the constraints of window shape or area, for usually, many turns per row fit in a window, and $n \to \infty$. For few large wires fitting a constraining window, k_p will decrease, as the Indiana General plot of k_p showed. Because high k_p is desirable, a conflict arises between it and using multiple strands of smaller wire to reduce the skin effect in magnetics design.

For machine winding, more regular and uniform layers result in a higher k_p. For these cases, it might be advisable to use a k_p weighted more closely to the maximum value instead of midway between extremes, as done here.

Conductor radius = $r_c(awg) := (4.10 \cdot mm) \cdot 2^{-\frac{awg}{6}}$

Insulated wire radius = $r(awg) := r_c(awg) + \sqrt{\left(1.568 \cdot 10^{-3} \cdot cm\right) \cdot r_c(awg)}$ triple insulation

Wire packing factor = $A_{conductor}/A_W = k_p$.

$$k_p(awg) := \frac{7}{8} \cdot \left(\frac{\pi}{2 \cdot \sqrt{3}}\right) \cdot \left(\frac{1}{1 + \sqrt{\frac{1.568 \cdot 10^{-3} \cdot cm}{r_c(awg)}}}\right)^2$$

$$AWG(r) := -19.93 \log\left(\frac{r}{4.10 \cdot mm}\right)$$

$$AWG_A(Area) := -9.97 \cdot \log\left(\frac{Area}{53.48 \, mm^2}\right)$$

Gage := 5 .. 40

Winding packing (fill) factor vs American Wire Gage