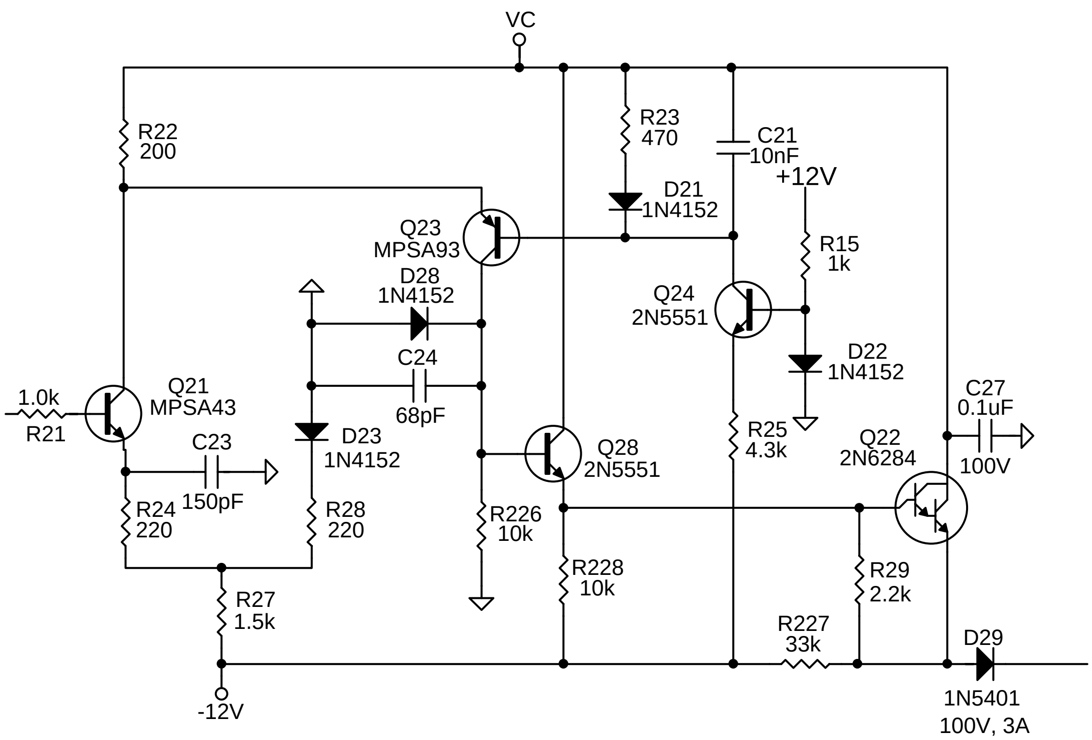
Why Circuits Oscillate Spuriously, Part 2: Amplifiers
by Dennis L Feucht

In Part 1 of this article we examined spurious oscillations caused by parasitic reactances that form resonant circuits, and also resonances caused by impedance gyrations in the high-frequency (hf) region of operation of active devices. This second cause is not well known in the industry, but should be. This article continues with a practical example of how hf gyration theory can be applied to a pulse amplifier design. Then we consider how amplifier hf output inductance can resonate with output capacitance.

Pulse Amplifier Output Stage

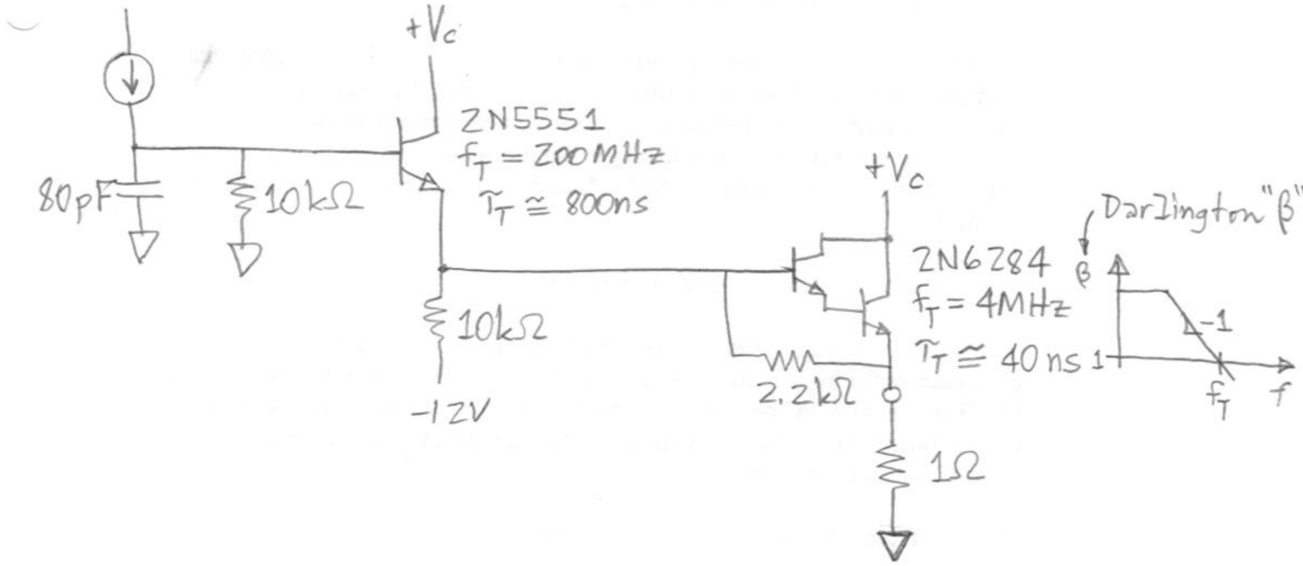
The output stages of a pulse amplifier used to drive loads with pulses of several amperes, is shown below.



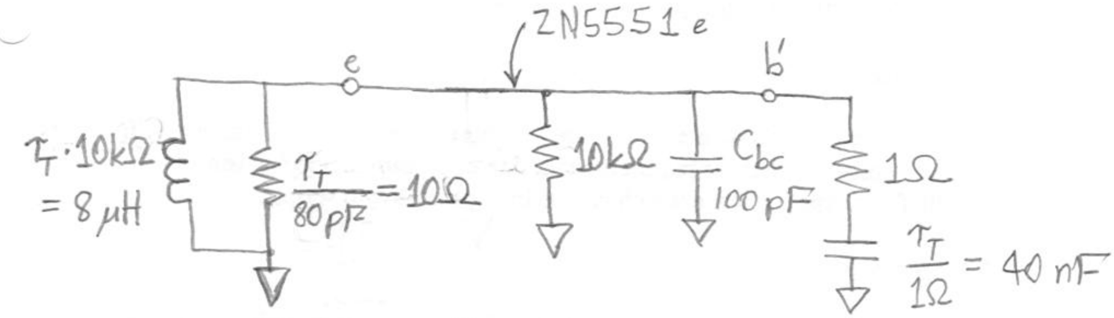
As part of the dynamic response analysis, the node at the emitter of Q28 is modeled in the hf region. The Darlington output stage, Q22, can be treated as a single, high-β BJT that presents a loading to Q28 due to the 1 Ω load (not shown). This load is referred to the base of Q22 using the hf model for R developed in Part 1 of this article. At the base emitter impedances are gyrated –90°. A resistance is thus gyrated into a capacitance of τ_T/R_E. The 2N6284 has an f_T = 4 MHz and thus a $\tau_T = 1/2\cdot\pi\cdot f_T \cong 40$ ns. Then the gyrated load resistance appears at the Q22 base as a capacitance of 40 nF in series with R_E = 1 Ω, as shown on my engineering project notebook page below. Q28 base-node capacitance is the sum of C24 and the semiconductor reverse-biased junctions of D28, C_{bc}(Q28) and C_{bc}(Q23), approximated as 80 pF. At the base node of Q28 is also a shunt load resistance of the previous complementary cascode stage. Both base R and C are referred to the emitter in the hf region by gyrating them +90°, as shown in the hf model on the notebook page. Consequently, C_B = 80 pF gyrates to a resistance of t_T/C_B = 800 ps/80 pF = 10 Ω. And for R_B the gyrated inductance is $\tau_T\cdot R_B$ = 8 μH. The hf model includes the Q28 emitter resistance of 10 kΩ and C_{bc}(Q22) = 100 pF.

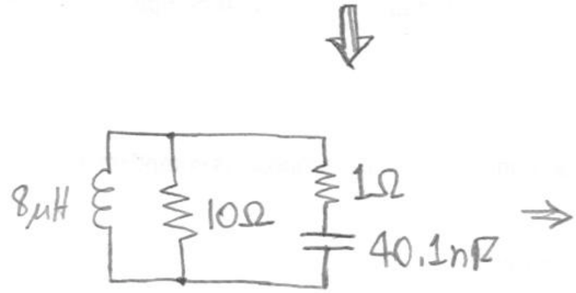
When all of this is combined and the resonant impedance and frequency are calculated, as shown, we can conclude that both parallel and series resonances are overdamped; the shunt resistance of 10 Ω < Z_n as is the series resistance of 15 Ω. Consequently, for a 1 Ω load, hf resonance should not cause spurious interstage oscillation in this section of the amplifier. This pulse amplifier was designed to drive pyrotechnic igniters, some of which can have resistances as low as 0.1 Ω. That would increase the output C by ten times and reduce Z_n to 4.5 Ω. Then the resonance is somewhat underdamped. The resonant frequency lies below the hf range of Q28 at 89 kHz, but not for Q22.

PG Output Stage Oscillation



High-frequency model:





$$Z_n = \sqrt{\frac{L}{C}} = 14\Omega$$

$$\tau_n = \sqrt{LC} = 567\,ns$$

$$f_n = \frac{1}{2\pi \cdot \tau_n} = 281\,kHz$$

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Output Impedance of a Feedback Amplifier

Finally, hf modeling can be extended to feedback amplifiers having a dominant single-pole response. A single-pole amplifier has an open-loop voltage gain of:

$$G = \frac{K}{s\tau_{bw} + 1}$$

where, $1/\tau_{bw}$ is the small-signal open-loop bandwidth. In the lf region, the open-loop output resistance, r_{out}, is reduced by feedback by a factor of $1 + GH$. The resulting closed-loop output impedance is:

$$Z_{out}(\text{cl}) = \frac{r_{out}}{1 + GH} = \frac{r_{out}}{1 + KH} \cdot \frac{s\tau_{bw} + 1}{s(\tau_{bw}/(1 + KH)) + 1}$$

where, KH is the static (dc) loop gain, a constant. This equation can be expressed as:

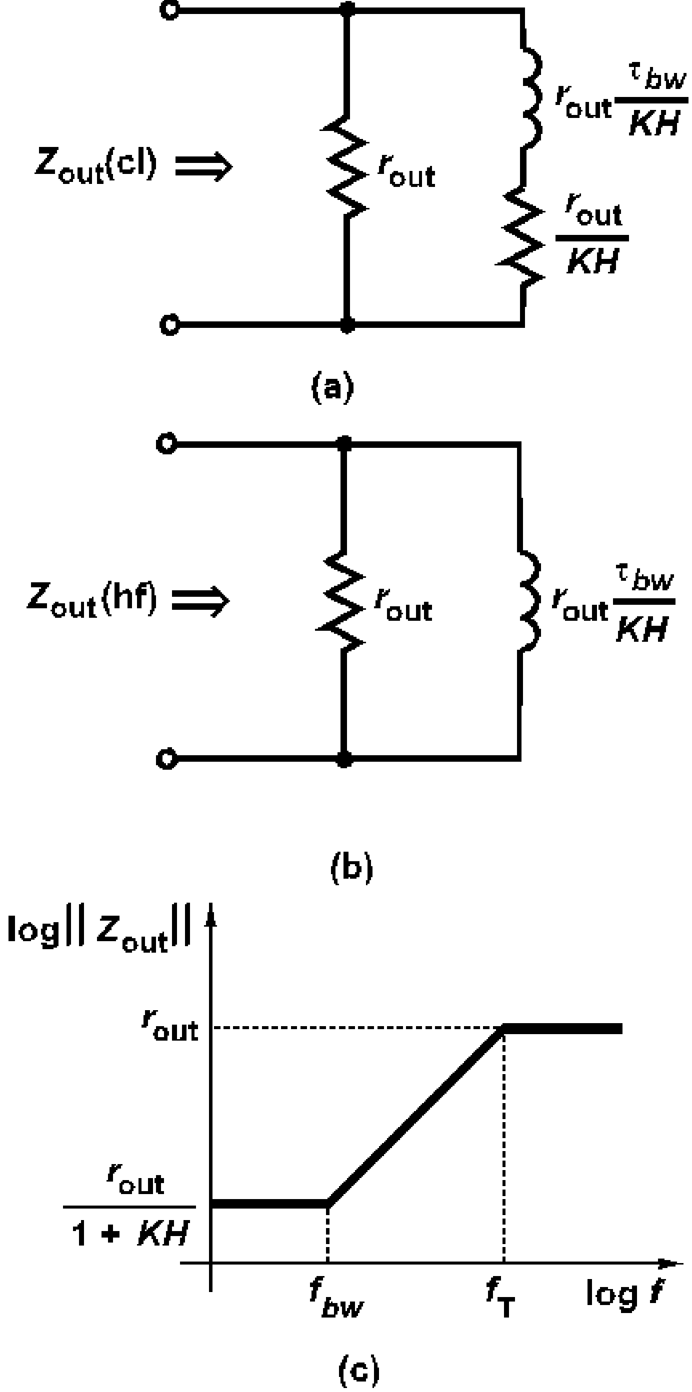
$$Z_{out}(\text{cl}) = \cfrac{1}{\cfrac{1}{r_{out}} + \cfrac{1}{s\tau_{bw}(r_{out}/KH) + (r_{out}/KH)}}$$

In continued-fraction form, the corresponding topology of Z_{out}(cl) is explicit. The complete model is shown below in (a), the hf model in (b), and the impedance magnitude versus frequency plotted in (c).

The lf closed-loop resistance, $r_{out}/(1 + KH)$, gyrates +90° at the open-loop bandwidth to appear inductive out to the unity-gain frequency:

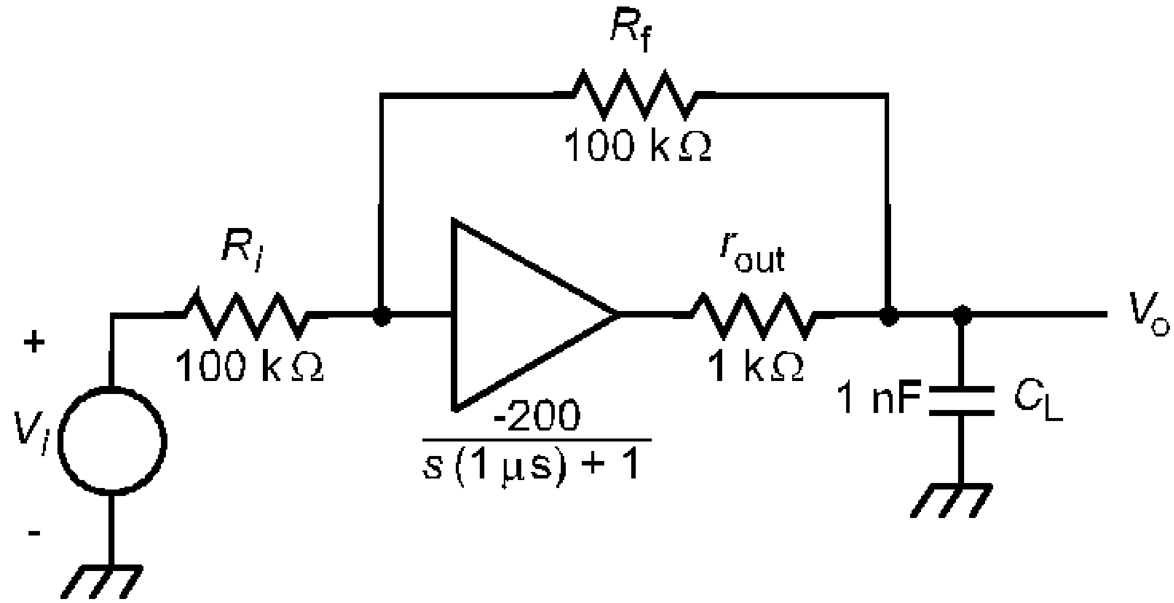
$$f_{bw}(1 + KH)$$

Above this unity-gain frequency, Z_{out}(cl) reverts to r_{out}. By analogy f_{bw} corresponds to the BJT f_β, the unity-gain frequency to f_T, and KH to β_o. The simplified hf equivalent output is derived by letting $KH \to \infty$ as $f \to 0$, with resulting output impedance corresponding to $Z_e(R_B)$. When r_{out} is generalized to Z_{out}, the corresponding BJT models readily apply. The hf equivalent circuit of Z_{out}(cl), as with the BJT model, is only valid above f_{bw}.



Zout(cl)
rout
rout τbw/KH
rout/KH
(a)
Zout(hf)
rout
rout τbw/KH
(b)
log|| Zout||
rout
rout/(1 + KH)
fbw
fT
log f
(c)

Example: Feedback Amplifier Output Resonance



The high-voltage amplifier shown here has a dc gain of –200 and a pole at 1 Ms^{-1} (τ_{bw} = 1 µs), or about 159 kHz. It has an output resistance of 1 kΩ and a load capacitance of 1 nF.

The loop gain KH = 100 and amplifier unity-gain frequency, $f_T \cong$ 15.9 MHz. Then τ_T = 10 ns, and the gyrated resistance is (10 ns)·(1 kΩ) = 10 µH and Z_n = 100 Ω. For a parallel resonance, the damping ratio:

$$\zeta = Z_n/2r_{out} = 100\ \Omega/2\ \mathrm{k}\Omega = 0.05$$

Then fractional step overshoot, M_p = 0.85. The simulated circuit $M_p \cong 0.71$, indicating that the hf model estimate of ζ is low. The amplifier simulations for loop gains of 10, 100, and 1000 are tabulated as follows:

KH	**1000**	**100**	**10**
M_p(SPICE)	0.87	0.71	0.25
L	1 µH	10 µH	100 µH
Z_n	31.6 Ω	100 Ω	316 Ω
r_{out}/KH	1 Ω	10 Ω	100 Ω
ζ (hf)	0.0158	0.050	0.158 = $Z_n/2r_{out}$ = $(r_{out}/KH)/2Z_n$
M_p (hf)	0.95	0.85	0.60
ζ	0.0316	0.10	0.316 = $1/\sqrt{KH}$
M_p	0.91	0.73	0.35

The table reveals that the predicted ζ using either pure parallel or series resonance is always low and the error increases as KH decreases. If r_{out}/KH and the load capacitance, C_L, are included in the model, then the expression for Z_{out}(cl) is:

$$Z_{out}(\mathrm{cl}) = \left(\frac{r_{out}}{1+KH}\right)\frac{s\tau_{bw}+1}{\{s[\tau_{bw}/(1+KH)]+1\}(sr_{out}C_L+1)}$$

and

$$\zeta = \frac{1}{\sqrt{KH}}$$

With this more accurate ζ (the lower entry in the above table) the agreement with simulation results is much better in the corresponding M_p values. This expression for resonant Z_{out}(cl) has the same form as Z_e, from which analogies can be made.

Even with the more exact ζ, the error grows with decreasing KH. This is due to the growing error in the asymptotic approximations of the impedance plot. For $KH = 10$, the error in M_p is quite apparent (40%); but for $KH = 100$, is much reduced (3%). In this example, also, the resonant frequency is near the center of the hf range thus reducing error due to linear approximation. Near either f_{bw} or f_T, this error becomes large; the approximate calculations of ζ should be used as a worst-case lower bound.

A note about project notebooks: Each time I start a new project I get a new three-ring binder notebook for my design documentation. I recommend the practice, though you might nowadays want to open a new project folder on your computer instead. For the latter, be sure to back it up on a USB Flash memory, or even a CD-RW if you dare. For the paper medium, initial and date every notebook page so that you can reconstruct what you did temporally at a later time. Then when you puzzle over how you arrived at a certain detail in the design, you can follow your original thinking chronologically from your notes.

In my case the above notebook page was originally dated 5JUN95, but I quickly found obvious calculation errors and had to redo it. However, the page recorded the path my 1995 thinking would have been led down by the arithmetic errors. The design has successfully been in production for years without oscillation problems. This corresponds with the corrected results, which do not look too bad. Keep in mind that in the full (lf & hf) model, Z_π appears in series, adding additional damping to the underdamped amplifier stage with 0.1 Ω load. Additionally, though not covered here, this is a current-source output (current-feedback) amplifier. If you do the analysis for a tranconductance amplifier (i_o/v_i), you will find that the hf gyration of its output resistance has a different result than for the voltage amplifier, as analyzed in the second section above, with its inductive hf output impedance. That section was taken from my book on analog circuit design: see http://www.innovatia.com for a description of it.

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