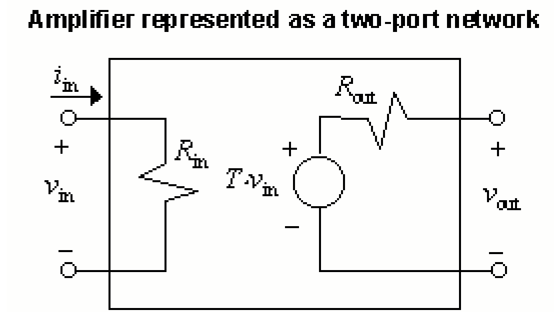
Transistor And Inductive-Switch Analogs

by Dennis Feucht

The transistor and the inductive switch, the core circuit of power converters, have interesting analogs. Both have three terminals (hence three configurations), are active devices, and have a key parameter. For bipolar junction transistors (BJTs) it is α; for the switch cell, it is duty-ratio, D. While α is fixed for BJTs, it can be varied for inductive switches, which leads to time-variant circuit behavior in switching converters. In this article, the two kinds of active devices are compared.

Transistor and Inductive Switch Configurations

A two-port network, such as an amplifier or power converter, has an input and an output port. Each port has a pair of terminals, as shown below.



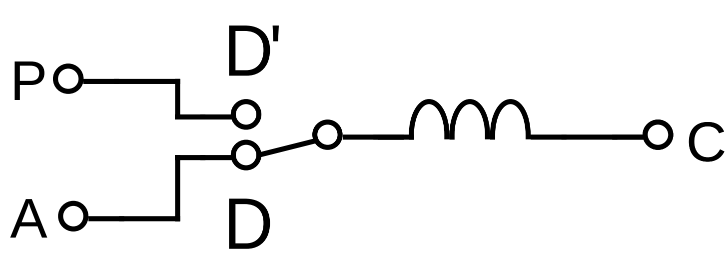
The relationship between output and input ports is usually expressed as a *transfer function* or *transmittance*. A device with three terminals, such as a transistor, has one input, one output, and one common terminal. The common terminal is shared by input and output ports, usually as a common ground terminal. This results in three configurations where each terminal assumes the common position. For the BJT and FET (either JFET or MOSFET) of either polarity (npn or pnp, n-channel or p-channel), corresponding configurations are listed in the following table:

common emitter	CE	common-source	CS
common-base	CB	common-gate	CG
common-collector (emitter follower)	CC	common-drain (source-follower)	CD

The analog between emitter-source, base-gate, and collector-drain is familiar and not hard to recognize, given the similarity in the devices.

Inductive-Switch Configurations

The inductive switch is a circuit network that can be regarded as a circuit element or *cell*, shown below:



The inductor is in series with a single-pole, double-throw current switch. It is in the *active* position (connected to the A terminal) for $D \cdot T_s$ of the time, where T_s is the switching period and D is the duty ratio. The switch is in the passive position for $D' \cdot T_s = (1 - D) \cdot T_s$.

This switch cell can be regarded as a three-terminal active device, like the transistor. Consequently, it too has three configurations: They are, by name:

- common passive (CP) or *buck*
- common active (CA) or *boost*
- common inductor (CI) or *buck-boost*

The voltage transfer function for a buck converter is:

$$\frac{V_s}{V_g} = D$$

where, $V_s \cong V_o$ is the secondary voltage (output voltage plus diode drop, if any) and V_g is the input voltage.

The transfer function is easily derived from flux balance of the inductor, so that on-time and off-time flux changes are equal, or

$$\Delta\lambda_{on} = V_g \cdot D \cdot T_s = \Delta\lambda_{off} = V_s \cdot (1 - D) \cdot T_s$$

In the CP configuration, the A terminal of the switch-cell is common to both V_g (input) and V_s (output) circuit loops.

The other two configurations have the following voltage transfer functions:

$$\frac{V_s}{V_g} = \frac{1}{1 - D}\text{, CA}$$

$$\frac{V_s}{V_g} = \frac{D}{1 - D}\text{, CI}$$

The current transfer function (output i/input i) of a BJT of the CE configuration (i_C / i_B) is β, for the CB it is α, and is $\beta + 1$ for the CC. Expressing β in terms of α:

$$\beta = \frac{\alpha}{1 - \alpha}$$

and:

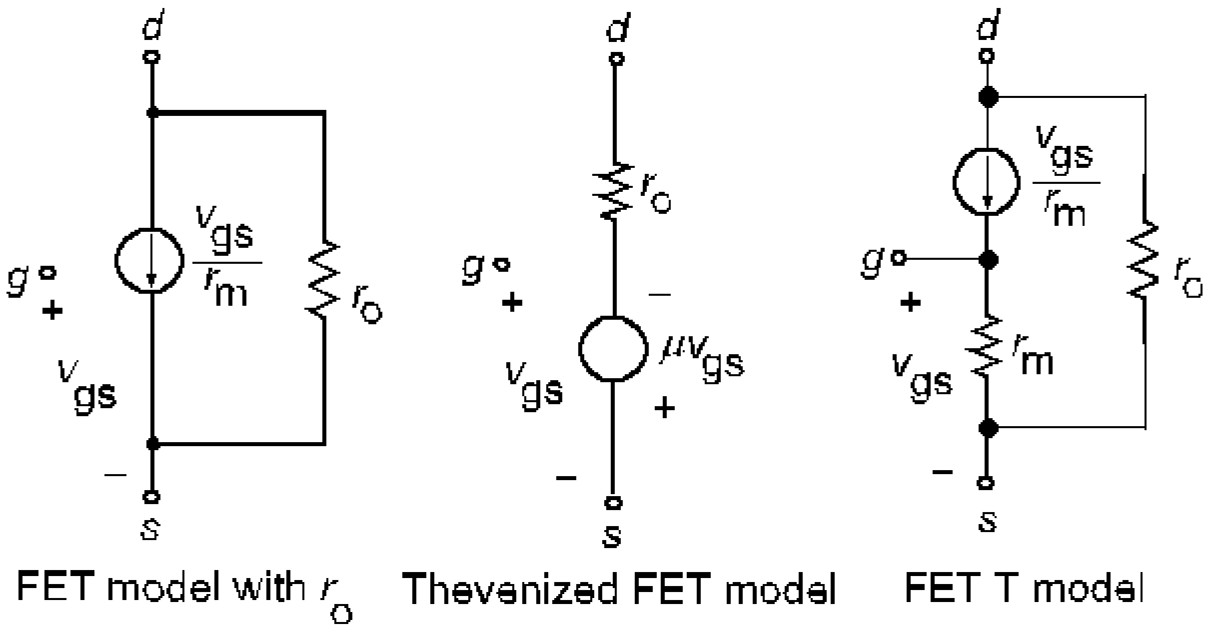
$$\beta + 1 = \frac{1}{1 - \alpha}$$

Then a correspondence between α and duty ratio, D, becomes evident, as summarized in the following table for BJTs and inductive switches. FETs have been added, resulting in a single, complete table:

BJT			FET			Inductive Switch		
CE	CB	CC	CS	CD	CG	CI	CP	CA
β $= \alpha/(1-\alpha)$	α	$\beta+1$ $= 1/(1-\alpha)$	μ $= \lambda/(1-\lambda)$	λ	$\mu+1$ $= 1/(1-\lambda)$	D/(1-D)	D	1/(1-D)

This $\alpha \leftrightarrow D$ analogy for BJTs applies to FETs as a $\lambda \leftrightarrow D$ analogy, though the correspondence may be less familiar. Just as β is a current-ratio parameter of BJTs, μ is the corresponding parameter for FETs, and is a voltage-ratio parameter:

$$\mu = -\frac{v_{DS}}{v_{GS}},\ i_{DS} = 0\text{ A}$$



The simple physical model of a FET is a dependent voltage source between source and drain (negative terminal to drain) for which the voltage is $\mu \cdot v_{GS}$, in series with a drain resistance, r_o. The alternative expression of this parameter has a correspondence with α, and is (like α to β, also the previous Greek letter) λ:

$$\lambda = \frac{\mu}{\mu + 1}$$

The ratio of source-to-gate voltage is the voltage gain of the CD (source-follower) configuration, and is λ. And the source-to-drain voltage gain is $\mu + 1$.

The table relates BJTs, FETs, and switches in their three configurations. The configurations are analogous:

- CE ↔ CS ↔ CI
- CB ↔ CD ↔ CP
- CC ↔ CG ↔ CA

Consequently, the terminals correspond too:

- emitter ↔ source ↔ inductor (common)
- base ↔ drain ↔ passive
- collector ↔ gate ↔ active

The one "imperfection" in the analogy is that the corresponding BJT-FET terminals of base-gate and collector-drain are reversed. This is due to the use of a current analogy for BJTs and the dual, a voltage analogy, for FETs. If either a β-based or μ-based model were used for either BJT or FET, then the BJT and

FET terminals would again correspond, but the analogous switch-cell terminals would correspond according to the β or μ analogies (for both BJT and FET) as shown in the table.

Closure

While these analogies may not allow immediate understanding of switch cells from transistor concepts, they can help analog engineers to think analogically about converter circuits. Just as transistor amplifier gain and port resistances vary with α, in converters they also vary with D. Duty ratio is best thought of as a switch-cell *parameter*, not a dynamic variable, though it may be varied dynamically in converters as a control variable.

This suggests why deceptively simple-looking converter circuits are actually more complicated and demanding of a designer than linearized analog circuits; imagine designing circuits for which β were varied. Just as the BJT transistor has a linear model based on constant β and r_e (at constant I_E), the inductive switch can be linearized around a constant-D operating point. Richard Tymerski first worked out this switch model in the mid-1980s, along with Vatché Vorperian (both at VPI at the time), and it can be used to do linear frequency-response analysis of converter (or any switched-inductor) circuits.