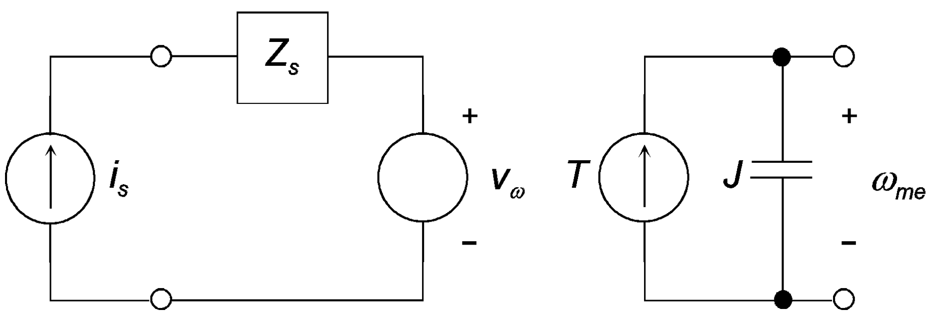
Piezoelectric Transducer Electromechanical Model
by Dennis L Feucht

Piezoelectric devices change shape when subjected to an electric field. They are emerging in importance in molecular mechanics (as positioners), in medical diagnostic and surgical equipment, and for vibration or noise suppression. As a particular kind of energy converter or transducer -- translational, electric (or electrostatic) machines -- they can be modeled using basic electromechanics theory, as presented for instance in *Electromechanical Motion Devices*, by Paul C Krause and Oleg Wasynczuk (McGraw-Hill, 1989). While the familiar magnetic motor, with energy stored in a magnetic field, dominates electric-machines textbooks, Krause and Wasynczuk also present the dual theory of machines with their energy stored in electric fields.

We start here with the more familiar magnetic machine model and work toward a circuit model of the solely electric machine. This model can then be used in design to predict and analyze the behavior of driver amplifiers and the motion control aspects.

Electromagnetic Model

The model of a field-oriented (that is, phase-controlled, with orthogonal *d* and *q* axes) synchronous motor is shown below, where Z_s is a series *RL*.



The electrical side of the machine is on the left; the mechanical side is modeled on the right, using the torque-current (or speed-voltage) analogy, for which *across* quantities (ω_{me}, v_ω) and *through* quantities (T, i_s) correspond analogously. *J* is the rotational inertia of the rotor and is analogously a capacitance. For a rotational magnetic motor, the electrical and mechanical variables are linked through one parameter, the circuit-referred flux in a mechanical reference frame, or flux-linkage λ_{me}:

$$T = \lambda_{me} \cdot i_s$$
$$v_\omega = \lambda_{me} \cdot \omega_{me},$$

where T = torque, i_s = stator current, v_ω = winding induced voltage, and ω_{me} = mechanical rotor speed.

For a translational (or linear) electromagnetic motor, the radius, *r*, which is embedded in *T* and ω_{me}, is extracted and cancelled to result in the translational force and speed equations:

$$F = \left(\frac{V}{U}\right)_{me} \cdot i$$
$$v_u = \left(\frac{V}{U}\right)_{me} \cdot u,$$

where $(V/U)_{me}$, a voltage parameter divided by a speed parameter, takes the place of λ_{me}. Then v_u is the induced voltage, and *u* is the mechanical speed. Mechanical impedances can be referred to the electrical side of the model (or vice versa) through the transducing parameter. First, for the rotational machine:

$$Z_{el} = \frac{v_\omega}{i_s} = \lambda_{me}^2 \cdot Z_{me},$$

where mechanical impedance:

$$Z_{me} = \frac{\omega_{me}}{T}$$

For the translational machine:

$$Z_{el} = \frac{v_u}{i} = \left(\frac{V}{U}\right)_{me}^2 \cdot Z_{me},$$

where

$$Z_{me} = \frac{u}{F}$$

The mechanical elements (such as rotational inertia, *J*), modeled analogously as electrical elements (such as capacitance), can be referred to the electrical side of the motor as electrical circuit elements by means of the above impedance transformation. Mechanical impedances are multiplied by conversion parameter, $(V/U)^2$, to become electrical circuit elements.

Piezoelectric Machine Model

Piezoelectric transducers are the duals of permanent-magnet motors. Instead of producing force from the interaction of magnetic fields, they are electrostatic machines that produce force from electric-field interaction. The physical relationships linking electrical and mechanical sides are usually given in the literature as a parameter designated d_{ij} where *i* and *j* indicate the dimensions orthogonal to that to which *d* applies. For *d* oriented through the thickness dimension, d_{33} is used. It relates mechanical strain, ε, to electric-field intensity, *E*:

$$\varepsilon = d \cdot E,$$

where $\varepsilon = \Delta L/L$; $\Delta L = x$ is the change in distance and *L* is the length of the transducer dimension stressed. This can be expressed more explicitly as:

$$\frac{x}{L} = d \cdot \frac{v}{L}$$

or

$$x = d \cdot v = \frac{1}{E_{me}} \cdot v$$

where E_{me} is now $1/d$, expressed as a conversion parameter. This can be rewritten as:

$$v = E_{me} \cdot x$$

Just as the magnetic machine has two electrical-mechanical conversion relations, the second corresponding equation is the electric component of the Lorentz force equation:

$$F = q \cdot E,$$

where *q* is charge.

Applied to the piezoelectric transducer, the second basic conversion equation results:

$$F = E_{me} \cdot q$$

These two equations relate electrical quantities *v* and *q* to mechanical quantities *x* and *F*. For both mechanical and electrical dynamic analysis, *q* and *x* are not the desired quantities; *i* and *u* are needed instead. The above equations can be differentiated, resulting in the appearance of *i* and *u*.

$$\dot{F} = E_{me} \cdot i$$
$$\dot{v} = E_{me} \cdot u$$

However, the quantities on the left sides of the equations are now no longer the ones desired!

It is possible to derive equations in *F*, *u*, *v*, and *i* by expressing *q* as:

$$q = C \cdot v = C \cdot (E_{me} \cdot x) = (C \cdot E_{me}) \cdot x$$

and differentiating:

$$i = (C \cdot E_{me}) \cdot u$$

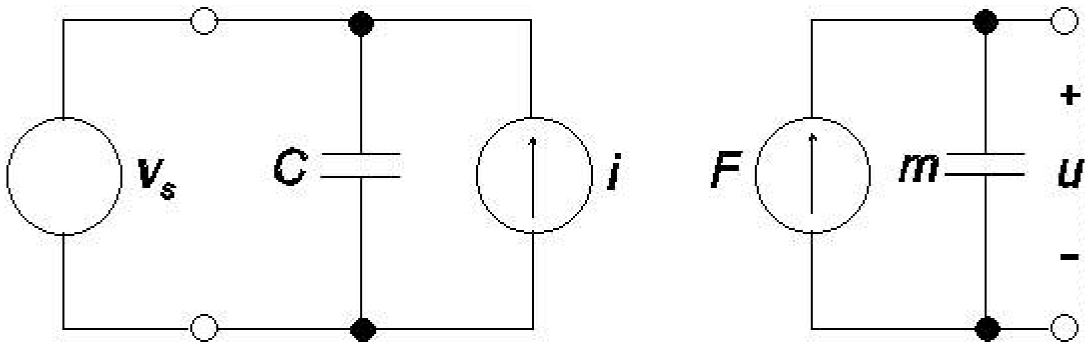
The second conversion equation is derived by substituting for *q* in the force equation:

$$F = E_{me} \cdot q = E_{me} \cdot (C \cdot v)$$

and

$$F = (C \cdot E_{me}) \cdot v.$$

where the common electromechanical parameter is $(C \cdot E_{me})$. [This can be expressed in the form of (*I/U*), the dual of the magnetic parameter, (*V/U*).] Consequently, the model is as shown below.



This model is not strictly the dual of the magnetic machine model because on the electrical side, the usually inconsequential parallel *R* has been omitted, and, more significantly, the mechanical side has retained the force-current analogy. (The force-voltage analogy is the mechanical dual.) Consequently, the conversion expression for the mechanical impedance, Z_{me}, will be inverted:

$$Z_{el} = \frac{v}{i} = \frac{F/(C \cdot E_{me})}{(C \cdot E_{me}) \cdot u} = \frac{1}{(C \cdot E_{me})^2} \cdot \frac{F}{u} = \frac{1}{(C \cdot E_{me})^2 \cdot Z_{me}}$$

That is a small price to pay for a conceptually more intuitive circuit model that is mechanically consistent with the magnetic-machine model. That is, by retaining the same electrical-mechanical elements (such as capacitance for mass or rotational inertia), it is not necessary to think about which of the dual analogies is being applied.

Closure

With this basic model, it is now possible to analyze circuits and systems containing piezoelectric machines as components. They appear to the power driver as a capacitive load, shunted by a current source. This source is analogous to the induced-voltage source of the magnetic-machine model. This displaced-current source transduces the mechanical speed by the conversion parameter ($C \cdot E_{me}$) and can be replaced by Z_{el}, the mechanical "circuit" referred to the electric side of the model according to the above equation.

In a magnetic-field motor the shaft inertia, modeled by a capacitance, will appear as a capacitance on the electrical side. Because of the inverse mechanical-impedance relation for Z_{el} (because the dual analog was not applied in the model), *m* appears as an inductive element on the electrical side, forming a parallel-resonant circuit with *C*. A corresponding resonance occurs in magnetic motors in that the series winding inductance forms a series-resonant circuit with the capacitance, which is the electrically-referred inertia. Consequently, the same kind of control challenges present themselves, and experience in the design of magnetic-motor motion controllers can be applied directly to piezoelectric machines.

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