Personal Energy Systems Calculations: Part I

by Dennis L Feucht

It has not been since the Carter presidency in the United States (1980) that energy has been the general concern it is now. The occasion of *peak oil* has not been an unexpected surprise. It had been predicted with some accuracy decades ago, and those predictions have held up fairly well. The need for new energy sources, or ways of living with much less energy, is emerging as a new emphasis for engineering. Power electronics appears to have a hopeful future.

The problem of alternative energy, especially when addressed at the small-scale (personal or family) level, where it is capable of solution by an engineer, can motivate some creative problem-solving. Some of the more obvious ideas for energy generation, or storage, can be analyzed at lunch doing math on napkins. Let's do some here. (Run off a copy of this article and take it to lunch with some other technologists.)

By putting numbers on some proposed solutions, their feasibility can be more readily assessed. Electric power companies bill customers for electric energy in units of kW·h. For our purposes, we will calculate in the more convenient metric unit of megajoules, MJ, where:

$$1\ \mathrm{kW \cdot h} = 3.6\ \mathrm{MJ}$$

A joule is a watt-second, which relates most directly to electrical units. Appliance energy ratings are typically given in the awkward unit of kW·h/year, which allows consumers to relate usage to their electric bills, where units are given in kW·h. For engineering purposes, average power in watts would be more useful; hence the conversion factor, using 8766 hours/year:

$$\frac{1\,\mathrm{kW \cdot h}}{\mathrm{yr}} \cdot \frac{1\,\mathrm{yr}}{8766\,\mathrm{h}} = 0.1141\,\mathrm{W}$$

or,

$$\frac{8.766\,\mathrm{kW \cdot h/yr}}{\mathrm{W}}$$

A refrigerator rated at 877 kW·h/yr consumes an average power of 100 W.

Energy Storage

We will start with the problem of energy storage. Two categories are chemical and mechanical. For chemical, the *hydrogen economy* might well be replaced by the *ethanol economy* considering the relative ease of storing and transporting the relatively safe ethanol. The problem for which ethanol is suggested is driven by the need for *mobile* storage, mainly for vehicles. We'll look at it for household energy later.

For stationary storage, one solution that comes to mind is mechanical: suspend a mass in a gravitational field, as a raised weight. How much energy can be stored by merely lifting weights? Electric motors can efficiently raise weights, and the scheme has a certain appealing simplicity. So let's do some calculations.

The energy of a mass, m, in a gravitational field with constant, g, at a raised height of h is:

$$W = m \cdot g \cdot h$$

We'll start with basic units of mass and height to become *calibrated*, with 1 kg raised 1 m:

$$W_0 = (1\,\mathrm{kg}) \cdot (9.8\,\mathrm{N/kg}) \cdot (1\,\mathrm{m}) = 9.8\,\mathrm{N} \cdot \mathrm{m} = 9.8\,\mathrm{J}$$

where, N is newtons (kg·m/s^2), the metric unit of force, and g is the acceleration of earth gravity, 9.81 m/s^2. W_0 is not much energy at all. To store 1 kW·h, or 3.6 MJ, at 1 m would require a mass of:

$$m_1 = \frac{3.6\,\mathrm{MJ}}{W_0} \cdot (1\,\mathrm{kg}) = 367.3\,\mathrm{Mg} = 183.7\ \mathrm{tonnes}$$

A metric ton (or *tonne*) is 1 Mg = 1000 kg, about 2200 pounds, 10% more than an English ton. How large is 184 tonnes? Five-ton trucks are rather large, yet small compared to this weight.

Consider the requirement for a single family, assuming an average power of 1 kW (a North American usage) delivered from storage for 24 hours, or 500 W for 48 hours (elsewhere). Throw in a few extra kW·h to account for inverter efficiency. Then:

27.78 kW·h = 100 MJ

To store 100 MJ, our working number for residential, single-family storage, would require a weight raised 1 m and have a mass of 5103 tonnes, a truly gargantuan amount of stuff. Trading distance for weight, a 510 tonne mass suspended 10 m has the same potential energy. The problem is that a thirty foot structure supporting 510 tonnes is not likely to catch on for backyard energy storage. Perhaps as the central core of a tall two-story house?

If translational weight is unattractive, consider rotational-mass energy storage, or flywheels. The relevant equation is:

$$W = J \cdot \omega$$

where, the angular speed is ω and the rotational inertia is:

$$J = m \cdot r_g^2$$

m is the flywheel mass and r_g the radius of gyration, the equivalent radius for J if all its mass were concentrated entirely in a ring at radius r_g.

We start again with unit quantities for mass and radius, with a mechanical frequency of 100 Hz. Then:

$$W = J \cdot \omega = m \cdot r_g^2 \cdot \omega = (1\,\mathrm{kg}) \cdot (1\,\mathrm{m})^2 \cdot (2\pi) \cdot (100\,\mathrm{Hz}) = 628\,\mathrm{J}$$

A flywheel weighing 1 kg (2.20 lb) with a 1 m radius (6.56 ft diameter) spinning at 6000 rpm stores little energy.

Let's now push the dimensions to realistic limits for a residential storage device by increasing the mass to a still-manageable 100 kg, keep the compact radius of 1 m, and spin it at 10 kHz. Then the storage capacity increases to:

$$W = (100\,\mathrm{kg}) \cdot (1\,\mathrm{m})^2 \cdot (2\pi) \cdot (10\,\mathrm{kHz}) = 6.28\,\mathrm{MJ}$$

This amount falls far short of our goal of 100 MJ, but is encouraging. The flywheel, though, about the weight of a 5 kW diesel generator, is roughly the same size. Because of the fast rotational speed of that much inertia, the unit will have to be confined to a safe place, such as an underground bunker, in case the flywheel loses its bearings. This has happened in flywheel laboratories at such places as MIT. An energetic flywheel can completely destroy a laboratory room, its structure, and everything in it. Happily, there have been no flywheel researcher casualties (that I am aware of).

Note that the energy storage increases by the square of the radius and only linearly with mass and speed. This suggests increasing the radius for increased storage. A 1 kg mass with a 10 m radius will have the same storage capacity as the previous instance. Because we have not yet attained our storage goal, a 10 m radius with a larger mass of 16 kg at the same 10 kHz speed will provide 100 MJ of storage. Implementation will require a large swimming-pool-sized hole in the back yard to install. It will not be inexpensive.

In view of the benefit of increasing radius, consider the energy of a 1000 kg (1 tonne) mass rotating at only 185.2 μHz with a radius of 6600 km. These are the values for an object in low-earth orbit, with an orbital period of 1.5 hours, given earth's radius as 6366 km. The speed of the object is 7.68 m/s. The energy *stored* in this object is:

$$W = (1000\text{ kg}) \cdot (6600\text{ km})^2 \cdot (2\pi) \cdot (185.2\times10^{-6}\text{ Hz}) = 50.68\times10^6\text{ MJ} = 14.08\text{ MW} \cdot \text{h}$$

This is sufficient storage for a city of over a half-million people. (With industrial loads factored in, it is perhaps closer to 200,000, or a city the size of Des Moines, Iowa.) Despite the slow rotational speed, the huge radius results in huge energy storage.

Flywheel storage has possibilities, but is not a clearly superior potential alternative to batteries. With magnetic bearings, perhaps the maintenance period for flywheels will be twice that of properly-maintained batteries, or about 15 to 20 years. With mechanical bearings, they are probably comparable. Deep-cycle 12 V lead-acid batteries of typical automobile size (225 A·h) have a storage capacity of about 2.7 kW·h or 9.72 MJ. For the 100 MJ storage goal, only a dozen are needed, including compensation for typical inverter inefficiency.

Energy Reduction

Another approach to the problem is to reduce energy need. This does not solve, but only prolongs the problem of energy generation. However, that can be part of what is needed to find a good solution: more time. By reducing single-household electricity needs to basics, the following energy budget results:

Load	**Average Power, W**
Refrigerator with freezing compartment	50
CFL Lights	46.5
Water pump	3.5
Modest air conditioning, fans	150
Equipment (consumer electronics, everything else)	250
Total average power	500 W = 12.0 kW·h/day

For a North American household, this is modest energy use. What is typical is at least twice that, or 1 kW. However, 500 W is achievable with little serious alteration in lifestyle (so I suppose). In developing countries, 500 W is more typical for urban residents.

Of equal significance is industrial demand. The use of field-oriented motor-drives for induction motors would make a considerable impact, though like replacement of incandescent lighting with fluorescent tubes (most efficient) or CFL lamps, it only delays the inevitable. White LEDs have an efficiency comparable to 100 W incandescent bulbs. Incandescent lamps are more efficient in larger sizes. A 100 W incandescent bulb has about a 6 % efficiency.

In Part II, we will go through the numbers for electric power from streams and small-scale ethanol production.

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