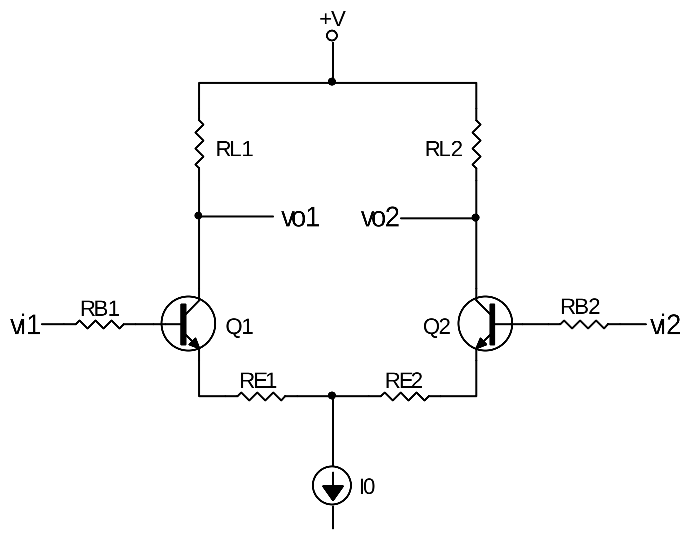
Temperature Compensation Of BJT Differential Amplifiers
by Dennis L Feucht

The bipolar junction transistor (BJT) emitter-coupled differential-pair circuit is a familiar amplifier stage in the repertoire of analog designers, but has a surprising obscurity that needs to be revealed. This TechNote examines the emitter-circuit current source, I_0, of BJT diff-amps and the effects on amplifier gain of different implementations for it.

The widespread belief that a BJT current source can temperature-compensate the diff-amp is true, but the conditions for it do not appear to be widely known, based on most designs. The typical circuit is shown below.



This is a differential-input, differential-output voltage amplifier. Both input and output quantities are differential, and the incremental gain of the circuit is:

$$A_v = \frac{v_o}{v_i} = \frac{v_{o2} - v_{o1}}{v_{i2} - v_{i1}} = \frac{A_{v2} \cdot v_{i2} - A_{v1} \cdot v_{i1}}{v_{i2} - v_{i1}} = \left.\frac{v_{o2}}{v_{i2}}\right|_{v_{i1}=0} - \left.\frac{v_{o1}}{v_{i1}}\right|_{v_{i2}=0} =$$

$$= -\alpha_2 \cdot \frac{R_{L2}}{r_{e1} + r_{e2} + R_{E1} + R_{E2} + R_{B1}/(\beta_1 + 1) + R_{B2}/(\beta_2 + 1)}$$

$$- \left(-\alpha_1 \cdot \frac{R_{L1}}{r_{e1} + r_{e2} + R_{E1} + R_{E2} + R_{B1}/(\beta_1 + 1) + R_{B2}/(\beta_2 + 1)} \right)$$

The condition for differential amplification is that $A_{v1} = A_{v2}$. The circuit is made symmetrical for:

$R_E = R_{E1} = R_{E2}$; $R_B = R_{B1} = R_{B2}$; $R_L = R_{L1} = R_{L2}$; $\beta = \beta_1 = \beta_2$, Q_1, Q_2 matched

Then the gain simply becomes:

$$A_v = 2 \cdot A_{v1} = 2 \cdot A_{v2} = -\alpha \cdot \frac{R_L}{r_e + R_E + R_B/(\beta + 1)} = -\alpha \cdot \frac{R_L}{r_M}$$

A goal of good design is to make A_v a fixed value. The choice of resistors with a low temperature coefficient (TC) and sufficiently tight accuracy is one factor. This is usually easy to achieve, though for high-precision design, the change in resistance due to change in ambient temperature is a factor to be considered. Even more so are *thermals*, changes in resistance due to changes in power dissipation with v_i. For very precise designs, the change in resistance with applied voltage must be considered too.

Other transistor parameters than the two (r_e and β) of the BJT T model used here - namely, r_o - also need to be included for precision design. We will assume that the BJTs have a sufficiently high Early voltage that r_o need not come into our list of considerations -- at least not here. In practice, this assumption is often valid.

BJTs are typically the least ideal elements of the circuit. From the gain formula, it is evident that two BJT parameters affect gain, the incremental emitter resistance, r_e, and β. For high β -- that is, for $\beta >> 1$ -- the gain factor,

$$\alpha = \frac{\beta}{\beta + 1}$$

approaches 1. For a typical β value of 200, then $\alpha = 0.995$, contributing a gain error of 0.5%. If that is too much error, α-compensation techniques are required. Usually, this error can be compensated by including it in the gain formula, as we have done. What is more important is how much it drifts with temperature. Typical TC(β) ≅ 1%/°C, then for large β, α has nearly a zero TC of around 50 ppm; α is not much of a problem.

The transresistance expression of A_v -- the denominator -- is the resistance across which the input voltage develops the common (emitter) current. The output current is modified by α, which accounts for loss along the way from the emitter circuit. This transresistance, r_M, also includes β in the R_B term. If R_B is kept small, and the inputs are driven by voltage sources, then this β is of no concern. If the sources are high in resistance, then the R_B term will affect gain by β variation with temperature. Its 1%/°C variation is scaled down by the extent to which $R_B/(\beta + 1)$ is not dominant in r_M. Thus, keeping this term negligible is another design factor.

The most troublesome term in r_M is r_e, for it varies with temperature and emitter current, I_E, according to:

$$r_e = \frac{V_T}{|I_E|} = \frac{kT/q_e}{|I_E|} \cong \frac{26\,\text{mV}}{|I_E|}, T = 300\,\text{K}$$

Thus r_e varies with the thermal voltage, V_T, which varies in proportion with absolute temperature:

$$\frac{dV_T}{dT} = \frac{V_T}{T}$$

At 300°K (about 80°F), this is 1/300°K or about 0.33%/°K = 0.33%/°C. For laboratory-quality instrument design, let us suppose the temperature range over which the equipment ought to be able to operate within its specifications is 15°C from room temperature, about 25°C ± 15°C, or 10°C to 40°C. Over a 15°C change from ambient, V_T changes about 5% -- far too much for most precision designs. Therefore, V_T variation in gain needs to be compensated.

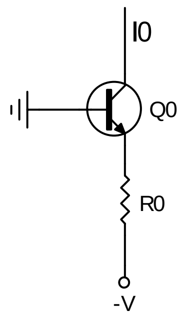
The simplest compensation for r_e is to make it a negligible term (along with the R_B term) in r_M. This is accomplished by making R_E dominant. For $R_E >> r_e$, the drift in r_e affects gain far less than 5%. In many cases, dominant external emitter resistance solves the drift problem, but at the expense of gain and power dissipation. By increasing I_0, then r_e is reduced proportionally, though circuit power increases. This is undesirable for power-limited equipment and it also exacerbates thermals by increasing $\Delta P_D(v_i)$ in the BJTs.

In some cases, r_e cannot be made negligible, and some compensation for it is desired. One of the most common schemes is to make I_0 track r_e and cancel its effect, at least approximately. To make I_0 have the TC of V_T, the simplest approach is to use a BJT current source implementation of I_0. The *b-e* junction voltage of the current-source BJT then decreases with temperature, I_0 increases, and decreases r_e.

Current-Source Circuits

The first circuit sourcing I_0 that we will consider is simply a resistor, R_0, returned to a negative supply. This *long-tailed* current source approaches an ideal current source as the supply voltage, $-V$, approaches negative infinity, along with the value of R_0. It does nothing to compensate for the TC of r_e.

The second implementation to consider is shown below.



This simple circuit has a voltage across R_0 of $V - V_{BE}$(Q0). As temperature, T, increases, V_{BE} decreases, but not with the TC of V_T. The other major BJT parameter affecting V_{BE} is the saturation current, I_S, as found in the *p-n* junction (*b-e* junction) voltage equation:

$$V_{BE} = V_T \cdot \ln\left(\frac{I_C}{I_S(T)}\right), I_C >> I_S$$

For a typical BJT, such as a PN3904, $I_S \approx 10^{-14}$ A. Then 1 mA of current produces a $V_{BE} \cong 0.65$ V.

Both V_T and I_S contribute to the TC(V_{BE}). I_S has a greater effect than V_T and of opposite polarity on V_{BE}, resulting in a combined effect of about –2 mV/°C for V_{BE}. (For more on BJT TC effects, see the volume, *Signal-Processing Circuits* of *Analog Circuit Design*, available at http://www.innovatia.com.) It is therefore more important to cancel I_S effects than those of V_T. Depending on the relative values of V and V_{BE}, the effect of the TC(V_{BE}) can be scaled by choice of R_E and supply voltage, V, which is often constrained by system-level design. By adding a resistor network between the emitter and ground, a Thévenin equivalent supply voltage and value of R_0 can be independently set. If scaled properly, as T increases, V_{BE} decreases and I_0 increases. If the increase is made to be such that the reduction in r_e due to it cancels the increase in r_e due to V_T, then r_e and gain remain constant.

The TC(r_e) is calculated as follows, by differentiating r_e with respect to T:

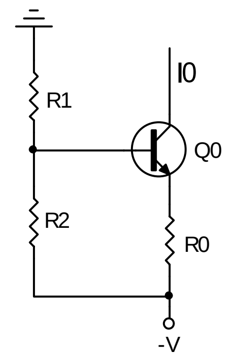
$$\frac{dr_e}{dT} = \frac{d}{dT}\left(\frac{V_T}{I_E}\right) = \left(\frac{1}{I_E}\right) \cdot \frac{dV_T}{dT} - \frac{V_T}{I_E^2} \cdot \frac{dI_E}{dT} = \frac{V_T}{I_E} \cdot \left(\frac{1}{V_T} \cdot \frac{dV_T}{dT}\right) - \frac{V_T}{I_E} \cdot \left(\frac{1}{I_E} \cdot \frac{dI_E}{dT}\right) = r_e \cdot [\mathrm{TC\%}(V_T) - \mathrm{TC\%}(I_E)]$$

where, TC% is the fractional change in TC.

The TC%(I_0) = TC%(I_E) can be determined as follows. The only change across R_0 is due to V_{BE}. Therefore, the fractional change in I_0 with T is:

$$\mathrm{TC\%}(I_0) = \frac{dI_0/dT}{I_0} = \frac{-(dV_{BE}/dT)/R_0}{I_0} = \frac{-dV_{BE}/dT}{V - V_{BE}} = \frac{+2\ \mathrm{mV/°C}}{V - V_{BE}}$$

For $TC\%(I_0) = TC\%(V_T) = 1/T \cong 0.33\%/°C$, and the voltage across R_0, $V - V_{BE} = 0.6$ V. With $-V = -1.25$ V, this is not an attractive compensation scheme. The polarity of $TC(I_0)$ is correct for compensation, but not its magnitude, which leads to the next scheme, shown below.



This implementation of I_0 is more versatile and more common in occurrence than the previous scheme. The base divider provides extra freedom for setting $TC\%(I_0)$ which, for ignored $TC(\beta)$, is now:

$$TC\%(I_0) = \frac{1}{I_0}\cdot\frac{dI_0}{dT} = \frac{1}{I_0}\cdot\frac{d}{dT}\left(\frac{\left(\frac{R_2}{R_1+R_2}\right)\cdot V - V_{BE}}{R_0 + (R_1 \| R_2)/(\beta+1)}\right) = \frac{1}{I_0}\cdot\frac{\left(-\frac{dV_{BE}}{dT}\right)}{R_0 + (R_1 \| R_2)/(\beta+1)}$$

$$= \frac{\left(-\frac{dV_{BE}}{dT}\right)}{\left(\frac{R_2}{R_1+R_2}\right)\cdot V - V_{BE}} = \frac{\frac{1}{V_{BE}}\cdot\left(-\frac{dV_{BE}}{dT}\right)}{\left(\frac{R_2}{R_1+R_2}\right)\cdot\frac{V}{V_{BE}} - 1} = \frac{-TC\%(V_{BE})}{\left(\frac{R_2}{R_1+R_2}\right)\cdot\frac{V}{V_{BE}} - 1}$$

The divider ratio that gives the correct compensation can now be found. When $TC\%(I_0)$ is set equal to $TC\%(V_T)$, then:

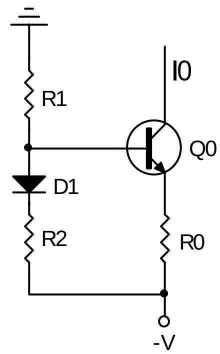
$$\left(\frac{R_2}{R_1+R_2}\right)\cdot\frac{V}{V_{BE}} = \frac{-TC\%(V_{BE})}{TC\%(V_T)} + 1 = \frac{+2\text{ mV/°C}/V_{BE}}{0.33\text{ \%/°C}} + 1 = \frac{0.6\text{ V} + V_{BE}}{V_{BE}}$$

or,

$$\frac{R_2}{R_1+R_2} = \frac{0.6\text{ V} + V_{BE}}{V} \cong \frac{1.25\text{ V}}{V}$$

for $V_{BE} = 0.65$ V. This result is interesting; whatever the value of V, the unloaded divider voltage must be 1.25 V for gain compensation. This is also the voltage of bandgap references, as well it should be. Bandgap circuits use the negative $TC(V_{BE})$ and scale it to cancel the positive $TC(V_T)$. When this is done the resulting bandgap voltage always comes out to be close to 1.25 V, and varies slightly with BJT doping levels.

Another current-source variation that is often used to provide rough temperature compensation is to insert a diode in series with R_2, as shown below.



The usual explanation is that the TC of the diode compensates for the TC of the BJT *b-e* junction, resulting in a more stable I_0. A typical example is to use a 1N4152 diode to compensate a PN3904. The diode and BJT *b-e* junctions are quite different, however. The diode doping levels are far less than even the BJT base, in order to achieve a higher breakdown voltage. The emitter minority carrier concentration is made intentionally large for good emitter injection efficiency into the base, at the expense of V_{BE} reverse breakdown, which is typically around 7 V, much below the 60 V of the diode. The point is that although both junctions are silicon, they are rather unmatched.

Suppose, however, that a similar BJT is used as a diode, with base connected to collector. Then the junction matching is better (though not as good as adjacent integrated BJTs), and allowing for $\alpha \cong 1$, then applying KVL around the BJT input loop:

$$V_T \cdot \ln\left(\frac{I_0}{I_D}\right) = I_D \cdot R_2 - I_0 \cdot R_0 \text{, or, } I_0 = \frac{I_D \cdot R_2 - V_T \cdot \ln(I_0 / I_D)}{R_0}$$

where, I_D is the diode current. If the junction currents are equal, the TC due to V_T is removed and the $\mathrm{TC\%}(I_0) \cong 0\%/°\mathrm{C}$. This is useful for applications where a stable current source is needed, but it does not compensate r_e of the diff-amp. The currents must deliberately be set unequal to achieve the desired TC, and for a compensating polarity of TC, it must be positive. Consequently, we must have $I_D > I_0$.

The $\mathrm{TC\%}(I_0)$ is found through implicit differentiation of I_0 in the above equation:

$$\mathrm{TC\%}(I_0) = \frac{1}{I_0} \cdot \frac{dI_0}{dT} = \frac{1}{I_0 \cdot R_0} \cdot \left(-V_T \cdot \frac{d}{dT} \ln\left(\frac{I_0}{I_D}\right) - \ln\left(\frac{I_0}{I_D}\right) \cdot \frac{dV_T}{dT}\right)$$

$$= -\frac{V_T}{I_0 \cdot R_0} \cdot \left(\mathrm{TC\%}(I_0) + \frac{1}{T} \cdot \ln\left(\frac{I_0}{I_D}\right)\right)$$

With additional algebraic manipulation:

$$\mathrm{TC\%}(I_0) = -\frac{(V_T / I_0 \cdot R_0) \cdot \ln(I_0 / I_D)}{T \cdot (1 + V_T / I_0 \cdot R_0)}$$

Then to compensate, set $\mathrm{TC\%}(I_0) = \mathrm{TC\%}(V_T) = 1/T$ and solve:

$$\frac{I_D}{I_0} = \exp\left(1 + \frac{I_0 \cdot R_0}{V_T}\right)$$

Because of the exponential function, practical current ratios require that the voltage across R_0 be not much larger than V_T. For $I_0 = 2$ mA, $R_0 = 22\ \Omega$, and $V_T = 26$ mV, then the voltage across R_0 is 44 mV, or $1.69{\cdot}V_T$, and $I_D = 14.77{\cdot}I_0 = 29.5$ mA, larger than is desired in most designs. Such small values of R_0 are required to keep R_0 from dominating the emitter circuit so that the TC of V_{BE} can be expressed. Yet in many designs, R_0 is relatively large and its voltage drop far exceeds V_T. As a consequence, the TC%(V_T) of r_e is not correctly compensated and a TC drift in gain exists.

The previous scheme, which omitted the base diode, was only slightly better in allowing for larger R_0 voltage. Perhaps we should go in the opposite direction and add a diode or two in the emitter. The TC of the combined junctions would be multiplied by the number of them, and that would allow R_E to be made proportionally larger. It is usually not desirable to add a large number of series diodes because the static (dc) stability of I_0 is not benefited. Consequently, use of the diff-amp current source to temperature-compensate r_e results in a circuit requiring careful static design. I_0 is then made sensitive to junction parameters, and these parameters, such as I_S, have a somewhat wide tolerance among discrete transistors, even of the same part number. Expect as much as 50 mV of variation among PN3904 BJTs at the same current and temperature. This compensation method is best suited for monolithic integration.

Closure

The widespread belief that a BJT current source can temperature-compensate a BJT diff-amp is true, but it often does not. Temperature compensation of I_0 for a constant r_e results in low voltage across the current-source external emitter resistance, R_0 -- so low that it can make accurate setting of I_0 infeasible.

Consequently, except for more elaborate schemes which amplify V_T, the dominant-R_E approach to diff-amp gain stability appears to still be the best.

Acknowledgement

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