Optimal State Estimation: Kalman, H_∞, and Nonlinear Approaches

by Dan Simon, Published by Wiley

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Review by Dennis L Feucht

As electronics becomes more sophisticated, control theory moves closer to the essential center of what an engineer must know to design leading-edge products or processes. Power electronics in particular is a fertile ground for the application of advanced control techniques. This book, by an EE professor and control theorist at Cleveland State University in Ohio and NASA consultant with extensive industrial experience, is a good place to begin in gaining an engineering-level understanding of the more settled areas of advanced control. In particular, one area of control that has grown into a sub-specialty has to do with the feedback of output and system state variables to the controller – the *H* block in the classic feedback diagram, as it were. In the presence of noise and an inability to observe as much of what is going on as desired, these techniques can be essential, and are referred to as *estimation*.

This book is loaded with matrix math, yet the style of presentation builds logically, one understandable step at a time. Simon has good didactic form, in trying to make sure that he is not assuming what the reader does not know but must in order to understand the flow of the presentation.

The introduction of the book shows which chapters one must understand to comprehend other chapters. The beginning chapters lay the basic theoretical foundation in linear systems theory. Chapter 1 contains a quick but well-presented survey of the later-used matrix math, how the basic linear system description is formulated, and then some numerical methods for its application on computers: rectangular, trapezoidal, and Runga-Kutta integration. Basic control concepts of observability, controllability, and stability are introduced.

Chapter 2 lays the groundwork of stochastic processes by covering probability theory, including multivariate statistics. Chapter 3 moves into the topic of the book title with least-squares estimation, first known to have been worked out around 1800 by *wunderkind* mathematician Karl F Gauss (when he was 18 years old). The author has made this chapter readable in spite of having to present alternate estimator forms such as (p. 87):

$$P_k = P_{k-1} - TP_{k-1}H_k^T S_k^{-1} H_k P_{k-1} - P_{k-1}H_k^T S_k^{-1} H_k P_{k-1} +$$
$$P_{k-1}H_k^T S_k^{-1} H_k P_{k-1} H_k^T P_{k-1} H_k^T S_k^{-1} H_k P_{k-1} + P_{k-1}H_k^T S_k^{-1} R_k S_k^{-1} H_k P_{k-1}$$

This is an intermediate matrix result in the derivation of a (happily) more simple formula. The significance of this is that the derivations are worked through in the book with enough detail to guide the reader through them. The author avoids hand-waving and attempts to teach the material in detail, expecting the reader to think through the derivations. This chapter delves into curve fitting before taking up Wiener filtering, followed by a few sections of filter optimization.

Chapter 4 prepares the way to Kalman filtering with the math for the propagation of system states, material needed for the state estimation algorithm which is the Kalman filter. The filter development is for discrete, sampled-data (discrete-time, continuous-value), and continuous systems. While presenting the math, the author also gives historical vignettes on those contributing to the development of the theory being presented. It fills in the overall picture in an understanding of state estimation. The problem sets at the end of chapters include both written and computer exercises.

Part II is "The Kalman Filter." Successive chapters cover the filter in its various formulations. Chapter 5 begins with the discrete-time filter and includes examples of its use which show what can go wrong in applying it. Analog circuits engineers will perk up at chapter. 8, “The continuous-time Kalman filter”, for the author writes:

The vast majority of Kalman filter applications are implemented on digital computers, so it may seem superfluous to discuss Kalman filtering for continuous-time measurements. However, there are still opportunities to implement Kalman filters in continuous time (ie in analog circuits) …steady-state continuous-time estimators can be analyzed using conventional frequency-domain concepts…

Various methods that reduce numerical-computation difficulties of the continuous-time Kalman filter are also presented, with some generalizations and examples.

Chapter 9, "Optimal smoothing," takes advantage of additional observation for estimation improvement. Then the book moves on to such engineering-oriented topics as how to verify the accuracy of a Kalman filter, reduced-order filters than do not observe all the system states, and robust filtering:

…although the Kalman filter is the optimal linear filter, it is not the optimal filter for non-Gaussian noise. Noise in nature is often approximately Gaussian but with heavier tails, and the Kalman filter can be modified to accommodate…

Such details, important for actual engineering implementation, are given.

Part III is "The H_∞ Filter." What is it? The Kalman filter requires an accurate system model to work right, but as the author explains: "… industry rarely has millions of dollars to spend on engineering problems (hence the inaccurate system models)." Engineers rarely have the statistics of the noise in industrial processes. The Kalman filter can be made robust, but the approaches are "somewhat ad-hoc in that they attempt to modify an existing approach. The H_∞ filter was specifically designed for robustness." (p. 334) This leads to constrained optimization using Lagrange multipliers, and a game-theory approach to deriving the discrete-time H_∞ filter, which minimizes the worst-case estimation error. This is in contrast to the Kalman filter's minimization of the expected value of the variance of the estimation error. Furthermore, the H_∞ filter does not make any assumptions about the statistics of the process and measurement noise (although this information can be used in the H_∞ filter if it is available).

An alternative derivation of the H_∞ filter is given using the familiar transfer-function approach. An additional chapter covers more on H_∞ filtering.

The final part of the book is on nonlinear filters, beginning with variations on extended Kalman filters. Chapter 15 presents "the particle filter," also called Monte Carlo filtering and survival of the fittest. It is not used more because of its intense computational requirement. This chapter discusses Bayesian state estimation, an approach that in mobile robotics has worked out very well. (See Carnegie-Mellon University robotics researcher Hans Moravec's uncertainty grid approach to world-map generation elsewhere.) The Kalman filter is optimal for linear, Gaussian-noise systems. For these systems, going from the Kalman filter to the particle filter increases computation but not accuracy. However, for nonlinear or non-Gaussian systems, the increased computation also increases estimation accuracy.

The book doesn't end here. The appendices are most interesting. Appendix A offers some brief history on the emergence of filtering, especially Kalman filtering. Appendix B is a commentary on other Kalman-filtering books. And Appendix C broadens the scope considerably. Titled "State estimation and the meaning of life," the author shows how a filter or state estimation explanation can be given to wider topics. Some of the subtitles are: "Forgiveness and noise suppression," "Discernment and bandwidth," "Fellowship and persistent excitation," "Spiritual growth and adaptive state estimation," "Spiritual perfection and estimator optimality," and "The one true way and the single best estimator." This appendix concludes by reference to Occam's Razor ("the simplest explanation is the most reasonable explanation") about which the author writes: "This idea is used in system identification to accept the simplest model structure that fits the observed data." He then briefly discusses whether having God in one's worldview simplifies or overly complicates it.

Simon has the knack of laying out what is inherently a fairly complicated mathematical subject-matter in a step-by-step, readable, and conceptually clear manner. Apart from the fact that Dan happens to be a good friend of mine, I am pleased to see that he also is an excellent conveyor of advanced control concepts. I give this book a high rating for anyone who wants to go beyond Bode and *s*-domain pole-zero plots, to venture into the increasingly-relevant domain of noisy linear (Kalman, H_∞) and nonlinear control – and a little beyond!